

3. Sia $V := \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^4 + z^2 \leq 1\}$. Calcolare

$$\int_V y \, dx \, dy \, dz, \quad \int_V |y| \, dx \, dy \, dz.$$

(a) $\int_V y \, dx \, dy \, dz = 0$ PER SIMMETRIA

(b) $\int_V |y| \, dx \, dy \, dz = 2 \int_{V^*} y \, dx \, dy \, dz =$

con $V^* = \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^4 + z^2 \leq 1 \wedge y \geq 0\}$

$$= 2 \int_0^1 y \, dy \int_0^{2\pi} d\theta \int_0^{\sqrt{1-y^4}} \rho \, d\rho =$$

$$= 2\pi \int_0^1 y \left[\rho^2 \right]_0^{\sqrt{1-y^4}} dy =$$

$$= 2\pi \int_0^1 y(1-y^4) \, dy = 2\pi \int_0^1 y - y^5 \, dy =$$

$$= 2\pi \left[\frac{y^2}{2} - \frac{y^6}{6} \right]_0^1 = 2\pi \left(\frac{1}{2} - \frac{1}{6} \right) = \frac{2}{3} \pi$$

