

$$z |z|^2 - iz \operatorname{Re}(z^*) + i = 0$$

MODO 1

$$z = a + ib$$

$$(a + ib)(a^2 + b^2) - (ai - b)a + i = 0$$

$$a^3 + ab^2 + iba^2 + ib^3 - a^2i + ab + i = 0$$

$$\begin{cases} a^3 + ab^2 + ab = 0 \\ b a^2 + b^3 + a^2 + 1 = 0 \end{cases} \quad \begin{cases} a(a^2 + b^2 + b) = 0 \\ b^3 + a^2(b+1) + 1 = 0 \end{cases} \rightarrow \begin{cases} a = 0 \\ a^2 + b^2 + b = 0 \end{cases}$$

$$\begin{cases} a = 0 \rightarrow b^3 + 1 = 0 & b = -1 \end{cases}$$

$$\begin{cases} a^2 = -b^2 - b \rightarrow b^3 - b(b+1)^2 + 1 = 0 & \cancel{b^3} - \cancel{b^3} - 2b^2 - b + 1 = 0 \end{cases}$$

$$b = \frac{-1 \pm 3}{4} \begin{cases} -1 \rightarrow a = 0 \\ 1/2 \rightarrow a \in \mathbb{C} \end{cases}$$

$$\rightarrow \underline{z = -i}$$

MODO 2

$$z |z|^2 - iz \operatorname{Re}(z^*) + i = 0$$

$$\begin{cases} i |z|^4 + |z|^2 \operatorname{Re}(z) - z^* = 0 \\ -i |z|^4 + |z|^2 \operatorname{Re}(z) - z = 0 \end{cases}$$

$$2 |z|^2 \operatorname{Re}(z) - 2 \operatorname{Re}(z) = 0$$

$$2 \operatorname{Re}(z) (|z|^2 - 1) = 0 \rightarrow \begin{cases} \operatorname{Re}(z) = 0 \\ |z|^2 = 1 \rightarrow |z| = 1 \end{cases}$$

$$1) \underline{\operatorname{Re}(z) = 0} \quad z = ib$$

$$z |z|^2 - iz \operatorname{Re}(z^*) + i = 0 \quad \cancel{b^3 i} + i = 0 \quad b = -1 \quad z = -i$$

$$2) \underline{|z|=1}$$

$$\begin{cases} i|z|^4 + |z|^2 \operatorname{Re}(z) - z^* = 0 \\ -i|z|^4 + |z|^2 \operatorname{Re}(z) - z = 0 \end{cases} \quad \text{DIFFERENZA}$$

$$2i|z|^4 - z^* + z = 0$$

$$2i + 2i \operatorname{Im}(z) = 0 \quad |z|^4 = 1 \quad \operatorname{Im}(z) = \frac{z - z^*}{2i}$$

$$\operatorname{Im}(z) = -1 \Rightarrow \operatorname{Re}(z) = 0 \Rightarrow z = -i$$

$$\leadsto \underline{z = -i}$$