BY GIMUSI 16/01/17

$$\lim_{x\to 0} x\cdot \operatorname{tg}\left(xa+\operatorname{arctg}\left(\frac{b}{x}\right)\right)$$

$b=0 \rightsquigarrow x\,\mathrm{TAN}(xa+\mathrm{ARCTAN}(b/x)) = x\,\mathrm{TAN}(xa) \to 0$

$b\neq 0$

$$x\,\mathrm{TAN}(xa+\mathrm{ARCTAN}(b/x)) = x\,\mathrm{TAN}\left(xa\pm\frac{\pi}{2}-\mathrm{ARCTAN}(x/b)\right) \overset{*}{=}$$

$$= x\,\mathrm{TAN}\left(xa\pm\frac{\pi}{2}-\frac{x}{b}+o(x)\right) =$$

$$= x\,\frac{\mathrm{SIN}\left(\pm\frac{\pi}{2}+xa-\frac{x}{b}+o(x)\right)}{\mathrm{COS}\left(\pm\frac{\pi}{2}+xa-\frac{x}{b}+o(x)\right)} = x\,\frac{\pm\mathrm{COS}\left(xa-\frac{x}{b}+o(x)\right)}{\mp\mathrm{SIN}\left(xa-\frac{x}{b}+o(x)\right)} =$$

$$= \underbrace{\frac{x}{xa-\frac{x}{b}+o(x)}}_{\to\, b/(ab-1),\ ab\neq 1}\ \underbrace{\frac{-\left(xa-\frac{x}{b}+o(x)\right)}{\mathrm{SIN}\left(xa-\frac{x}{b}+o(x)\right)}}_{\to -1}\ \underbrace{\mathrm{COS}\left(xa-\frac{x}{b}+o(x)\right)}_{\to 1} \to$$

$$\to \frac{b}{1-ab} \quad ab\neq 1$$

PER CONSIDERARE ANCHE IL CASO $ab=1$:

$$\overset{*}{\ldots} = x\,\mathrm{TAN}\left(xa\pm\frac{\pi}{2}-\frac{x}{b}+\frac{x^3}{3b^3}+o(x)\right) =$$

$$= x\,\frac{\mathrm{SIN}\left(\pm\frac{\pi}{2}+xa-\frac{x}{b}+\frac{x^3}{3b^3}+o(x)\right)}{\mathrm{COS}\left(\pm\frac{\pi}{2}+xa-\frac{x}{b}+\frac{x^3}{3b^3}+o(x)\right)} = x\,\frac{\pm\mathrm{COS}\left(xa-\frac{x}{b}+\frac{x^3}{3b^3}+o(x)\right)}{\mp\mathrm{SIN}\left(xa-\frac{x}{b}+\frac{x^3}{3b^3}+o(x)\right)} =$$

$$= \underbrace{\frac{x}{\cancel{xa}-\cancel{\frac{x}{b}}+\frac{x^3}{3b^3}+o(x)}}_{\to\, \frac{b}{|b|}\cdot(+\infty)}\ \underbrace{\frac{-\left(xa-\frac{x}{b}+\frac{x^3}{3b^3}+o(x)\right)}{\mathrm{SIN}\left(xa-\frac{x}{b}+\frac{x^3}{3b^3}+o(x)\right)}}_{\to -1}\ \underbrace{\mathrm{COS}\left(xa-\frac{x}{b}+\frac{x^3}{3b^3}+o(x)\right)}_{\to 1} \to$$

$$\to \frac{-b}{|b|}\cdot(+\infty)$$