BY GIMUSI 12/12/16

Data la matrice

$$A = \begin{pmatrix} -1 & 2/3 & 0 \\ 1 & 1/3 & 1/2 \\ 1 & 1/3 & -1/2 \end{pmatrix}$$

associata ad un endomorfismo, si vogliono trovare autovalori e autovettori.

$$|A-\lambda I| = \begin{vmatrix} -1-\lambda & 2/3 & 0 \\ 1 & 1/3-\lambda & 1/2 \\ 1 & 1/3 & -1/2-\lambda \end{vmatrix} =$$

$$= -(1+\lambda)\left[(1/3-\lambda)(-1/2-\lambda)-1/6\right] - 2/3\,(-1/2-\lambda \overset{-(1+\lambda)}{-1/2}) =$$

$$= -(1+\lambda)\,(-1/6-\lambda/3+\lambda/2+\lambda^2-1/6-2/3) =$$

$$= -(1+\lambda)\,(\lambda^2+\lambda/6-1) = 0 \implies \begin{cases} \lambda_1 = -1 \\ \lambda^2+\lambda/6-1 = 0 \end{cases}$$

$$\lambda_{2,3} = \frac{-1/6 \pm \sqrt{1/36+4}}{2} = -\frac{1}{12} \pm \frac{\sqrt{145}}{12}$$

$\underline{\lambda_1 = -1}$

$$(A-\lambda_1 I)x_1 = \begin{pmatrix} 0 & 2/3 & 0 \\ 1 & -2/3 & 1/2 \\ 1 & 1/3 & 1/2 \end{pmatrix} x_1 = 0$$

$$\implies x_1 = \begin{pmatrix} 1 \\ 0 \\ -2 \end{pmatrix}$$

$$\underline{\lambda_2 = -\frac{1}{12} + \frac{\sqrt{145}}{12}}$$

$$(A-\lambda_2 I)x_2 = \overset{B}{\begin{pmatrix} -\frac{11}{12}-\frac{\sqrt{145}}{12} & \frac{2}{3} & 0 \\ 1 & \frac{5}{12}-\frac{\sqrt{145}}{12} & \frac{1}{2} \\ 1 & \frac{1}{3} & -\frac{5}{12}-\frac{\sqrt{145}}{12} \end{pmatrix}} x_2 = 0$$

$$(B)_{1,i} \cdot X_2 = 0 \Rightarrow X_2 = \begin{pmatrix} 2/3 \\ \frac{11}{12}+\frac{\sqrt{145}}{12} \\ \alpha \end{pmatrix}$$

$$(B)_{2,i} \cdot X_2 = 0 \Rightarrow \frac{2}{3} + \left(\frac{5}{12} - \frac{\sqrt{145}}{12}\right)\left(\frac{11}{12} + \frac{\sqrt{145}}{12}\right) + \frac{\alpha}{2} = 0$$

$$\frac{2}{3} + \frac{55}{144} + \frac{5\sqrt{145}}{144} - \frac{11\sqrt{145}}{144} - \frac{145}{144} + \frac{\alpha}{2} = 0$$

$$\frac{\overset{6}{\overbrace{96+55-145}}}{144} - 6\,\frac{\sqrt{145}}{144} + \frac{\alpha}{2} = \frac{1}{24} - \frac{\sqrt{145}}{24} + \frac{\alpha}{2} = 0$$

$$\alpha = -\frac{1}{12} + \frac{\sqrt{145}}{12} \quad \Rightarrow \quad X_2 = \begin{pmatrix} 2/3 \\ \frac{11}{12}+\frac{\sqrt{145}}{12} \\ -\frac{1}{12}+\frac{\sqrt{145}}{12} \end{pmatrix}$$

$(B)_{3,i} \cdot X_2 \overset{?}{=} 0$ PER VERIFICA

$$\Rightarrow \frac{2}{3} + \frac{1}{3}\left(\frac{11}{12} + \frac{\sqrt{145}}{12}\right) + \left(-\frac{5}{12} - \frac{\sqrt{145}}{12}\right)\left(-\frac{1}{12} + \frac{\sqrt{145}}{12}\right) =$$

$$= \frac{2}{3} + \frac{11}{36} + \frac{\sqrt{145}}{36} + \frac{5}{144} - \frac{5\sqrt{145}}{144} + \frac{\sqrt{145}}{144} - \frac{145}{144} =$$

$$= \frac{96+44+5-145}{144} + \frac{4-5+1}{144}\sqrt{145} = 0$$

$$\underline{\lambda_3 = -\frac{1}{12} - \frac{\sqrt{145}}{12}}$$

C

$$(A - \lambda_3 I)x_3 = \begin{pmatrix} -\frac{11}{12} + \frac{\sqrt{145}}{12} & \frac{2}{3} & 0 \\ 1 & \frac{5}{12} + \frac{\sqrt{145}}{12} & \frac{1}{2} \\ 1 & \frac{1}{3} & -\frac{5}{12} + \frac{\sqrt{145}}{12} \end{pmatrix} x_3 = 0$$

$$(C)_{1,:} \cdot x_3 = 0 \implies x_3 = \begin{pmatrix} 2/3 \\ \frac{11}{12} - \frac{\sqrt{145}}{12} \\ \alpha \end{pmatrix}$$

$$(C)_{2,:} \cdot x_3 = 0 \implies \frac{2}{3} + \left(\frac{5}{12} + \frac{\sqrt{145}}{12}\right)\left(\frac{11}{12} - \frac{\sqrt{145}}{12}\right) + \frac{\alpha}{2} = 0$$

$$\frac{2}{3} + \frac{55}{144} - 5\frac{\sqrt{145}}{144} + 11\frac{\sqrt{145}}{144} - \frac{145}{144} + \frac{\alpha}{2} = 0$$

$$\frac{\overset{6}{96+55-145}}{144} + 6\frac{\sqrt{145}}{144} + \frac{\alpha}{2} = 0 \qquad \alpha = \frac{1}{12} + \frac{\sqrt{145}}{12}$$

$$\implies x_3 = \begin{pmatrix} 2/3 \\ \frac{11}{12} - \frac{\sqrt{145}}{12} \\ \frac{1}{12} + \frac{\sqrt{145}}{12} \end{pmatrix}$$

$$(C)_{3,:} \cdot x_3 \overset{?}{=} 0$$ PER VERIFICA

$\rightsquigarrow$ A È DIAGONALIZZABILE $(\lambda_1 \neq \lambda_2 \neq \lambda_3)$

$$M^{-1}AM = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -\frac{1}{12} + \frac{\sqrt{145}}{12} & 0 \\ 0 & 0 & -\frac{1}{12} - \frac{\sqrt{145}}{12} \end{pmatrix}$$

$$M = \begin{array}{c} \begin{array}{ccc} \qquad x_1 \qquad & \qquad x_2 \qquad & \qquad x_3 \qquad \\ \downarrow & \downarrow & \downarrow \end{array} \\ \begin{pmatrix} 1 & 2/3 & 2/3 \\ 0 & \frac{11}{12} + \frac{\sqrt{145}}{12} & \frac{11}{12} - \frac{\sqrt{145}}{12} \\ -2 & -\frac{1}{12} + \frac{\sqrt{145}}{12} & \frac{1}{12} + \frac{\sqrt{145}}{12} \end{pmatrix} \end{array}$$