BY GIMUSI 03/12/16

Per quali valori di $t \in \cancel{\mathbb{R}}$ la matrice simmetrica

$$\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 2 & t \end{bmatrix} = A$$

è associata, tramite la base canonica di $\mathbb{R}^3$, ad una applicazione bilineare simmetrica $g_t : \mathbb{R}^3 \times \mathbb{R}^3 \to \mathbb{R}$, di segnatura $(1, 2, 0)$?

SEGNATURA
$$\begin{cases} 1 \text{ AUTOVALORE POSITIVO } m_+ \\ 2 \text{ AUTOVALORI NEGATIVI } m_- \\ 0 \text{ AUTOVALORI NULLI } m_0 \end{cases}$$

MODO 1 – CALCOLO AUTOVALORI

$$|A - \lambda I| = \begin{vmatrix} -1-\lambda & 0 & 0 \\ 0 & 1-\lambda & 2 \\ 0 & 2 & t-\lambda \end{vmatrix} = -(1+\lambda)(1-\lambda)(t-\lambda) + 4(1+\lambda) =$$

$$= (1+\lambda)(-t + \lambda + \lambda t - \lambda^2 + 4) = 0 \qquad \begin{cases} \lambda = -1 \\ \lambda^2 - \lambda(1+t) + (t-4) = 0 \end{cases}$$

$$\lambda = \frac{1+t \pm \sqrt{(1+t)^2 - 4(t-4)}}{2} = \frac{1+t \pm \sqrt{t^2 - 2t + 17}}{2} =$$

$$= \frac{1+t \pm \sqrt{(t-1)^2 + 4^2}}{2} \quad ?$$

SCOMODO DA STUDIARE

MODO 2 – CARTESIO

$$|A - \lambda I| = (1+\lambda)(-t + \lambda + \lambda t - \lambda^2 + 4) =$$

$$= -t + \lambda + \cancel{\lambda t} - \cancel{\lambda^2} + 4 - \cancel{\lambda t} + \cancel{\lambda^2} + \lambda^2 t - \lambda^3 + 4\lambda =$$

$$= -\lambda^3 + \lambda^2 t + 5\lambda + (4 - t)$$

			m_0	m_+	$m_- = 3 - m_0 - m_+$
$t<0$	$- \,^{P} - \,^{V} + \,^{P} +$	$\rightsquigarrow$	0	1	2
$t=0$	$- \,^{V} + \,^{P} +$	$\rightsquigarrow$	0	1	2
$0<t<4$	$- \,^{V} + \,^{P} + \,^{P} +$	$\rightsquigarrow$	0	1	2
$t=4$	$- \,^{V} + \,^{P} +$	$\rightsquigarrow$	1	1	1
$t>4$	$- \,^{V} + \,^{P} + \,^{V} -$	$\rightsquigarrow$	0	2	1

MODO 3 – COMPLETAMENTO DEI QUADRATI

$$x^T A x = (x \; y \; z) \begin{pmatrix} -x \\ y+2z \\ 2y+\delta z \end{pmatrix} = -x^2 + y^2 + 2yz + 2yz + \delta z^2 =$$

$$= -x^2 + y^2 + \delta z^2 + 4yz = -x^2 + (y+2z)^2 - 4z^2 + \delta z^2 =$$

$$= -x^2 + (y+2z)^2 + (\delta - 4) z^2$$

	n_-	n_+	n_0		
$\delta > 4$	1	2	0 $\rightsquigarrow$	$q(v) = 0 \iff v = 0$	
$\delta = 4$	1	2	1 $\rightsquigarrow$	$q(v) = 0$	$v = (0, s, -s/2)$
$\delta < 4$	2	1	0 $\rightsquigarrow$	$q(v) = 0 \iff v = 0$	

MODO 4 – SYLVESTER

$$A = \begin{array}{ccc} 1 & 2 & 3 \\ \end{array}$$
$$A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 2 & \delta \end{bmatrix}$$

$DET_1 = -1 \qquad DET_2 = \begin{vmatrix} -1 & 0 \\ 0 & 1 \end{vmatrix} = -1$

$DET_3 = -1 \begin{vmatrix} 1 & 2 \\ 2 & \delta \end{vmatrix} = -(\delta - 4) = 4 - \delta$

			n_+	n_-	n_0
$\delta < 4$	$+ \overset{V}{} - \overset{P}{} - \overset{V}{} +$	$\rightsquigarrow$	1	2	0
$\delta = 4$	$+ \overset{V}{} - \overset{P}{} - \; 0$	$\rightsquigarrow$	1	1	1 $DET_3 = 0$
$\delta > 4$	$+ \overset{V}{} - \overset{P}{} - \overset{P}{} -$	$\rightsquigarrow$	2	1	0