BY GIMUSI 05/12/16

calcolo del limite di $\frac{1}{x}\log\frac{x}{\log(1+x)}$ in 0^+

$$\log(1+x) = x - \frac{1}{2}x^2 + o(x^2)$$

$$\frac{\log(1+x)}{x} = 1 - \frac{1}{2}x + o(x)$$

$$\frac{1}{x}\log\frac{x}{\log(1+x)} = -\frac{1}{x}\log\left(1 - \frac{1}{2}x + o(x)\right)$$

$$\log\left(1 - \frac{1}{2}x + o(x)\right) = -\frac{1}{2}x + o(x) + o\left(-\frac{1}{2}x + o(x)\right) =$$

$$o\left(\frac{1}{2}x + o(x)\right) = \left(-\frac{1}{2}x + o(x)\right)\cdot\omega(x) =$$

$$= x\left(-\frac{1}{2} + \frac{o(x)}{x}\right)\cdot\omega(x) = x\cdot\omega_1(x) = o(x)$$

$$= -\frac{1}{2}x + o(x)$$

$$\frac{1}{x}\log\frac{x}{\log(1+x)} = -\frac{1}{x}\log\left(1 - \frac{1}{2}x + o(x)\right) = \frac{1}{2} + \frac{o(x)}{x} \to \frac{1}{2}$$