Sia $[x]$ la parte intera di $x \in \mathbb{R}$. Per $x \geq \frac{1}{2}$ consideriamo la funzione
f definita da:

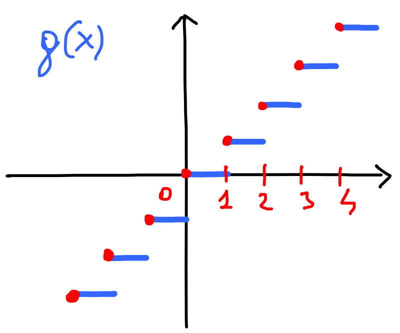
f(x) = $[x] + (x - [x])^{\frac{1}{2}}$.

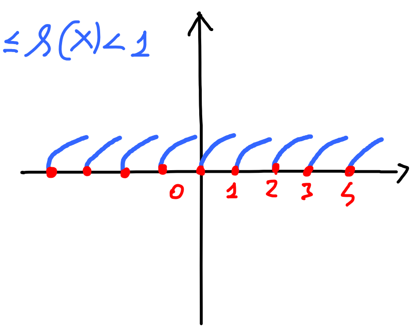
Devo dimostrare che f è continua Per $x \geq \frac{1}{2}$ e strettamente crescente su $[1, +\infty]$

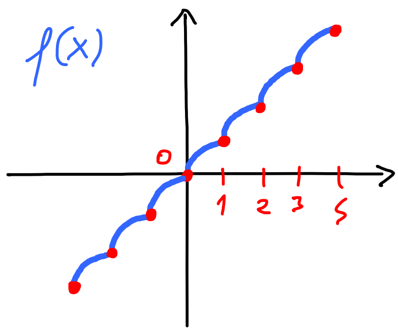
1) CONTINUITÀ

SI DEVE MOSTRARE CHE $\forall x_0 \geq \frac{1}{2} \quad \lim_{x \to x_0} f(x) = f(x_0)$

$$f(x) = g(x) + h(x) \qquad g(x) = [x] \qquad h(x) = (x - [x])^{1/2}$$







$g(x)$ E $h(x)$ SONO CONTINUE IN OGNI INTERVALLO $(n, n+1)$

$\forall n \in \mathbb{Z}$ $\Rightarrow$ (METATEOREMA) $f(x)$ È CONTINUA $\forall x \notin \mathbb{Z}$

$$\forall x_0 \in \mathbb{Z} \qquad \lim_{x \to x_0^+} f(x) = \lim_{x \to x_0^+} [x] + (x - [x])^{1/2} =$$

$$= x_0 + (x_0 - x_0)^{1/2} = x_0 = f(x_0)$$

$$\lim_{x \to x_0^-} f(x) = \lim_{x \to x_0^-} [x] + (x - [x])^{1/2} =$$

$$= x_0 - 1 + (x_0 - (x_0 - 1))^{1/2} = x_0 - 1 + 1 = x_0 = f(x_0)$$

$$\Rightarrow \lim_{x \to x_0} f(x) = f(x_0) \qquad \forall x_0 \in \mathbb{R}$$

2) MONOTONIA

SI DEVE MOSTRARE CHE

$$\forall x_1, x_2 \geq 1/2 \quad x_2 > x_1 \Rightarrow f(x_2) > f(x_1)$$

$$f(x_2) = [x_2] + (x_2 - [x_2])^{1/2} \overset{?}{>} [x_1] + (x_1 - [x_1])^{1/2}$$

(i) $[x_2] > [x_1]$

$$f(x_2) - f(x_1) = \overbrace{g(x_2) - g(x_1)}^{\geq 1} + \overbrace{h(x_2) - h(x_1)}^{\in (-1, 1)} > 0$$

(ii) $[x_2] = [x_1]$

$$f(x_2) - f(x_1) = \overbrace{g(x_2) - g(x_1)}^{= 0} + \overbrace{h(x_2) - h(x_1)}^{> 0} > 0$$

$\leadsto$ $f(x)$ È STRETT. CRESCENTE $\forall x \in \mathbb{R}$