BY GIMUSI 05/11/16

Funzioni trigonometriche inverse 2

Argomenti: principio di induzione ? **Difficoltà:** ⋆⋆⋆

Prerequisiti: principio di induzione

1. Completare la seguente tabella (se la quantità richiesta non ha senso, farlo presente!):

	a)		b)		c)
$\arcsin(\sin(1/2))$	1/2	$\arccos(\cos(1/2))$	1/2	$\arctan(\tan(1/2))$	1/2
$\arcsin(\sin 1)$	1	$\arccos(\cos 1)$	1	$\arctan(\tan 1)$	1
$\arcsin(\sin 2)$	π−2	$\arccos(\cos 2)$	2	$\arctan(\tan 2)$	2−π
$\arcsin(\sin 3)$	π−3	$\arccos(\cos 3)$	3	$\arctan(\tan 3)$	3−π
$\arcsin(\sin 4)$	π−4	$\arccos(\cos 4)$	2π−4	$\arctan(\tan 4)$	4−π
$\sin(\arcsin 1/2)$	1/2	$\cos(\arccos 1/2)$	1/2	$\tan(\arctan 1/2)$	1/2
$\sin(\arcsin 1)$	1	$\cos(\arccos 1)$	1	$\tan(\arctan 1)$	1
$\sin(\arcsin 2)$	—	$\cos(\arccos 2)$	—	$\tan(\arctan 2)$	2

2. (a) Disegnare i grafici delle funzioni (precisando in particolare cosa differenzia il primo dal terzo)

$$\arcsin(\sin x), \qquad \arccos(\cos x), \qquad \arctan(\tan x).$$

   (b) Disegnare i grafici delle funzioni

$$\sin(\arcsin x), \qquad \cos(\arccos x), \qquad \tan(\arctan x).$$

3. Nella seguente tabella si considerano alcune restrizioni delle usuali funzioni trigonometriche ad insiemi diversi da quelli standard. Si richiede di dimostrare che tali restrizioni danno luogo a funzioni invertibili e di determinare l'espressione delle funzioni inverse in termini delle funzioni trigonometriche inverse classiche.

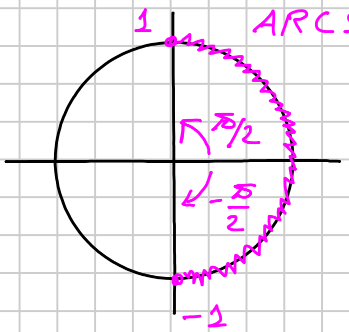
Funzione	Definizione	Partenza/Arrivo	Inversa
$\mathrm{mycos}\, x$	$\cos x$	$[8\pi, 9\pi] \to [-1, 1]$	ARCCOSX + 8π
$\mathrm{yourcos}\, x$	$\cos x$	$[9\pi, 10\pi] \to [-1, 1]$	−ARCCOSX + 10π
$\mathrm{hersin}\, x$	$\sin x$	$[7\pi/2, 9\pi/2] \to [-1, 1]$	ARCSINX + 4π
$\mathrm{hissin}\, x$	$\sin x$	$[9\pi/2, 11\pi/2] \to [-1, 1]$	−ARCSINX + 5π
$\mathrm{ourtan}\, x$	$\tan x$	$(15\pi/2, 17\pi/2) \to \mathbb{R}$	ARCTANX + 8π

1/5

1. Completare la seguente tabella (se la quantità richiesta non ha senso, farlo presente!):

	a)		b)		c)
$\arcsin(\sin(1/2))$	$1/2$	$\arccos(\cos(1/2))$	$1/2$	$\arctan(\tan(1/2))$	$1/2$
$\arcsin(\sin 1)$	1	$\arccos(\cos 1)$	1	$\arctan(\tan 1)$	1
$\arcsin(\sin 2)$	$\pi-2$	$\arccos(\cos 2)$	2	$\arctan(\tan 2)$	$2-\pi$
$\arcsin(\sin 3)$	$\pi-3$	$\arccos(\cos 3)$	3	$\arctan(\tan 3)$	$3-\pi$
$\arcsin(\sin 4)$	$\pi-4$	$\arccos(\cos 4)$	$2\pi-4$	$\arctan(\tan 4)$	$4-\pi$
$\sin(\arcsin 1/2)$	$1/2$	$\cos(\arccos 1/2)$	$1/2$	$\tan(\arctan 1/2)$	$1/2$
$\sin(\arcsin 1)$	1	$\cos(\arccos 1)$	1	$\tan(\arctan 1)$	1
$\sin(\arcsin 2)$	—	$\cos(\arccos 2)$	—	$\tan(\arctan 2)$	2

1.a)



$$\begin{cases} \text{ARCSIN}(\text{SIN}\,\alpha) = \alpha \iff -\frac{\pi}{2} \le \alpha \le \frac{\pi}{2} \\ \text{SIN}(\text{ARCSIN}\,\beta) = \beta \iff -1 \le \beta \le 1 \end{cases}$$

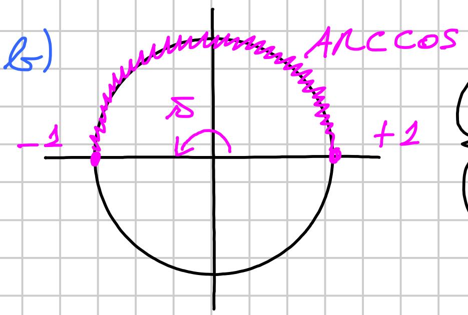
$\text{ARCSIN}(\text{SIN}\,2)$ $\qquad \text{SIN}\,2 = \text{SIN}\left(\frac{\pi}{2} - \left(2-\frac{\pi}{2}\right)\right) = \text{SIN}(\pi-2)$

$|\pi-2| \le \frac{\pi}{2}$ $\qquad \text{ARCSIN}(\text{SIN}\,2) = \text{ARCSIN}(\text{SIN}(\pi-2)) = \pi-2$

$\text{ARCSIN}(\text{SIN}\,3) = \text{ARCSIN}(\text{SIN}(\pi-3)) = \pi-3$

$\text{ARCSIN}(\text{SIN}\,4) = \text{ARCSIN}(\text{SIN}(\pi-4)) = \pi-4$

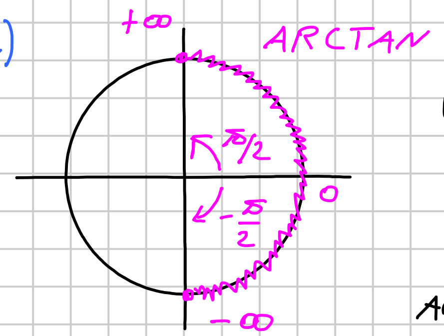
1.b)



$$\begin{cases} \text{ARCCOS}(\text{COS}\,\alpha) = \alpha \iff 0 \le \alpha \le \pi \\ \text{COS}(\text{ARCCOS}\,\beta) = \beta \iff -1 \le \beta \le 1 \end{cases}$$

$\text{COS}\,4 = \text{COS}(2\pi-4)$ $\qquad \text{ARCCOS}(\text{COS}\,4) = \text{ARCCOS}(\text{COS}(2\pi-4)) = 2\pi-4$

1.c)



$$\begin{cases} \text{ARCTAN}(\text{TAN}\,\alpha) = \alpha \iff -\frac{\pi}{2} < \alpha < \frac{\pi}{2} \\ \text{TAN}(\text{ARCTAN}\,\beta) = \beta \quad \forall \beta \in \mathbb{R} \end{cases}$$

$\text{ARCTAN}(\text{TAN}\,2) = \text{ARCTAN}(\text{TAN}(2-\pi)) = 2-\pi$

2. (a) Disegnare i grafici delle funzioni (precisando in particolare cosa differenzia il primo dal terzo)

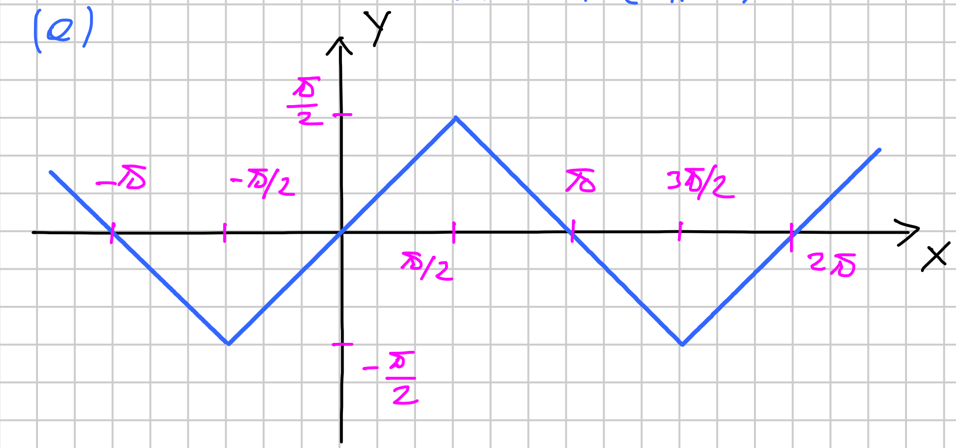
$$\arcsin(\sin x), \qquad \arccos(\cos x), \qquad \arctan(\tan x).$$

(b) Disegnare i grafici delle funzioni

$$\sin(\arcsin x), \qquad \cos(\arccos x), \qquad \tan(\arctan x).$$

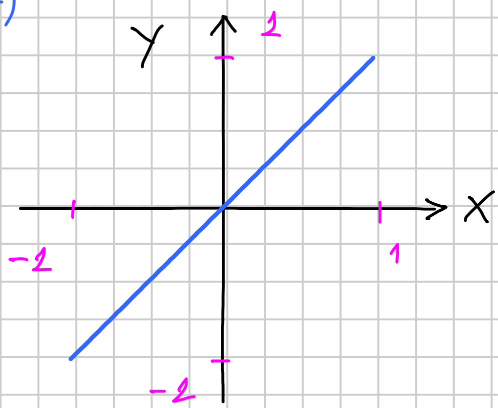
(a)

ARCSIN(SINX)

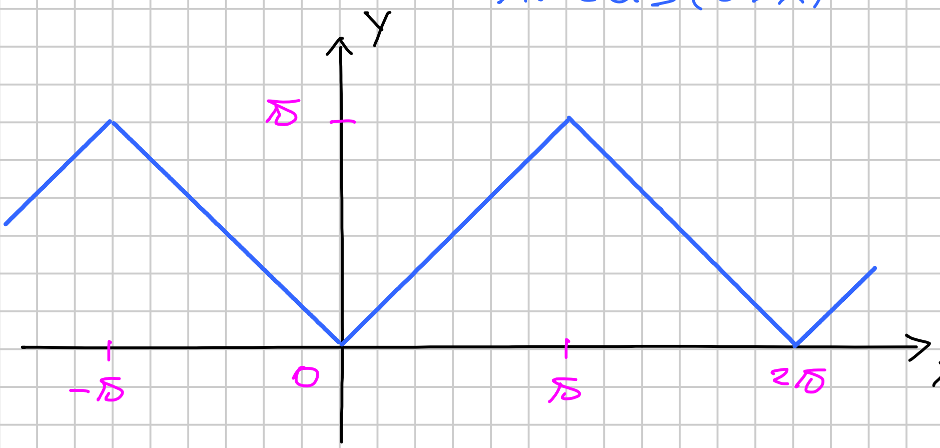


(b)

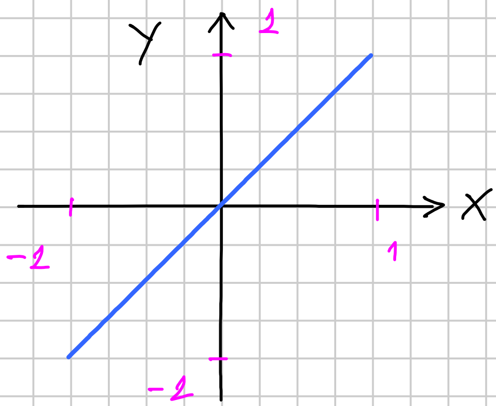
SIN(ARCSINX)



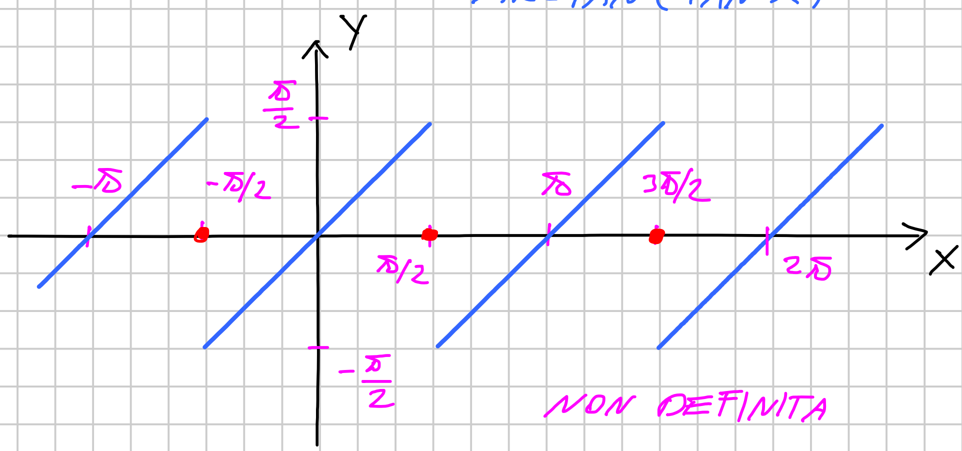
ARCCOS(COSX)



COS(ARCCOSX)

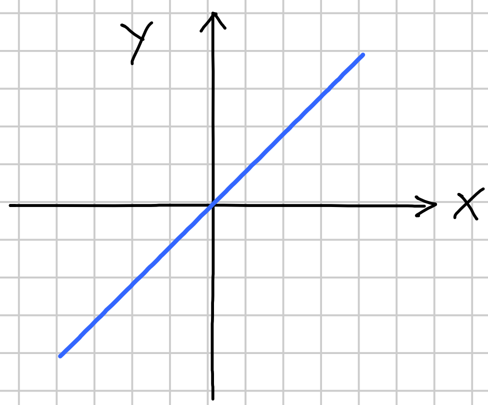


ARCTAN(TANX)



NON DEFINITA PER $x = n\frac{\pi}{2}$ $n \in \mathbb{Z}$

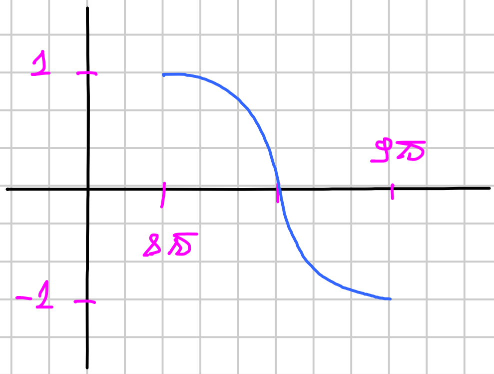
TAN(ARCTANX)=X



3. Nella seguente tabella si considerano alcune restrizioni delle usuali funzioni trigonometriche ad insiemi diversi da quelli standard. Si richiede di dimostrare che tali restrizioni danno luogo a funzioni invertibili e di determinare l'espressione delle funzioni inverse in termini delle funzioni trigonometriche inverse classiche.

	Funzione	Definizione	Partenza/Arrivo	Inversa
(a)	$\mathrm{mycos}\,x$	$\cos x$	$[8\pi, 9\pi] \to [-1, +1]$	ARCCOSX $+ 8\pi$
(b)	$\mathrm{yourcos}\,x$	$\cos x$	$[9\pi, 10\pi] \to [-1, +1]$	−ARCCOSX $+ 10\pi$
(c)	$\mathrm{hersin}\,x$	$\sin x$	$[7\pi/2, 9\pi/2] \to [-1, 1]$	ARCSINX $+ 5\pi$
(d)	$\mathrm{hissin}\,x$	$\sin x$	$[9\pi/2, 11\pi/2] \to [-1, 1]$	−ARCSINX $+ 5\pi$
(e)	$\mathrm{ourtan}\,x$	$\tan x$	$(15\pi/2, 17\pi/2) \to \mathbb{R}$	ARCTANX $+ 8\pi$

3a)



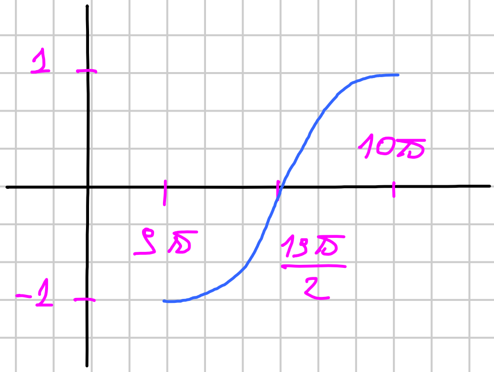
MYCOSX $[8\pi, 9\pi] \to [-1, 1]$

BIGETTIVA ~> ESISTE INVERSA

INVERSA:

ARCCOS(X) $+ 8\pi$

3b)



YOURCOSX $[9\pi, 10\pi] \to [-1, 1]$

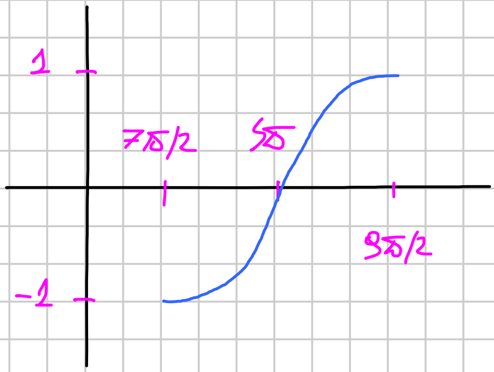
BIGETTIVA ~> ESISTE INVERSA

INVERSA:

−ARC COS(X) $+ 10\pi$ =

= ARC SIN(X) $+ \frac{19\pi}{2}$

3c)



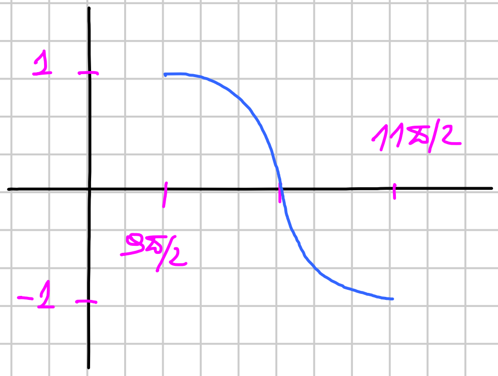
HERSINX $[7\pi/2, 9\pi/2] \to [-1, 1]$

BIGETTIVA ~> ESISTE INVERSA

INVERSA:

ARCSINX $+ 5\pi$

3d)



HISSINX $[9\pi/2, 11\pi/2] \rightarrow [-1, 1]$

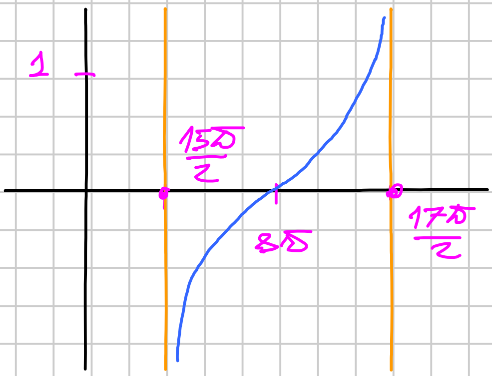
BIGETTIVA $\rightsquigarrow$ ESISTE INVERSA

INVERSA:

$-\arcsin x + 5\pi =$

$\arccos x + \frac{9\pi}{2}$

3e)



OURTANX $[15\pi/2, 17\pi/2] \rightarrow \mathbb{R}$

BIGETTIVA $\rightsquigarrow$ ESISTE INVERSA

INVERSA:

$\arctan x + 8\pi$