

Ricorrenza	Dato $x_0$	Monot	Limite	Inf	Sup
$x_{n+1} = \frac{x_n}{1+x_n^2}$	$-1/2$	$\uparrow$	$0$	$-1/2$	$0$

$$x_{n+1} = \frac{x_n}{1+x_n^2} \quad x_0 = -\frac{1}{2}$$

PIANO CON MONOTONIA

$$(i) \quad -\frac{1}{2} \leq x_n \leq 0 \quad \forall n \in \mathbb{N}$$

$$(ii) \quad x_{n+1} \geq x_n$$

$$(iii) \quad x_n \rightarrow l \in \mathbb{R}$$

$$(iv) \quad l = 0$$

DIM. (iii) SEGUE DA (i) E (ii) PER TEO. SUCC. MONOTONE

DIM. (iv)  $x_n \rightarrow l$

$$x_{n+1} = \frac{x_n}{1+x_n^2} \leadsto l = \frac{l}{1+l^2}$$

$$l(1+l^2) = l \quad \cancel{l+l^3=l} \quad l^3=0 \quad l=0$$

DIM. (ii)  $x_{n+1} \stackrel{?}{\geq} x_n \quad \frac{x_n}{1+x_n^2} \geq x_n \quad \cancel{x_n} \geq \cancel{x_n} + x_n^3$

$$x_n^3 \leq 0 \quad x_n \leq 0 \quad [\text{vd. p.to (i)}]$$

DIM (i) PER INDUZIONE

PASSO BASE  $n=0 \quad x_1 = \frac{-1/2}{1+1/4} \quad -1/2 \leq x_1 \leq 0 \quad \text{OK}$

PASSO INDUTTIVO  $H_p: -1/2 \leq x_n \leq 0 \quad \text{TESI: } -1/2 \leq x_{n+1} \leq 0$

$$-1/2 \leq x_n \leq 0 \quad -1/2 \leq \frac{-1/2}{1+x_n^2} \leq \frac{x_n}{1+x_n^2} \leq \frac{0}{1+x_n^2} = 0$$

$$\leadsto -1/2 \leq x_{n+1} \leq 0$$