BY GIMUSI 20/02/16

$$\lim_{x\to+\infty} \log(1+e^{x^2})\left(\cos\frac{1}{x}-1\right) \to -1/2$$

$$\cos\frac{1}{x} - 1 = -\frac{1}{2x^2} + o\left(\frac{1}{x^2}\right)$$

$$\lg(1+e^{x^2}) = \lg(1+e^{x^2}) - \lg(e^{x^2}) + \lg(e^{x^2}) =$$

$$= \log\left(1+\frac{1}{e^{x^2}}\right) + x^2 = x^2 + \frac{1}{e^{x^2}} + o\left(\frac{1}{e^{x^2}}\right)$$

$$\lg(1+e^{x^2})\left(\cos\frac{1}{x} - 1\right) =$$

$$= \left(x^2 + \frac{1}{e^{x^2}} + o\left(\frac{1}{e^{x^2}}\right)\right)\left(-\frac{1}{2x^2} + o\left(\frac{1}{x^2}\right)\right) =$$

$$= \left(1 + \frac{1}{x^2 e^{x^2}} + o\left(\frac{1}{x^2 e^{x^2}}\right)\right)\left(-\frac{1}{2} + o(1)\right) \to -\frac{1}{2}$$