Cambi di base 1

Argomenti: matrici di cambio di base **Difficoltà:** ⋆⋆⋆

Prerequisiti: cambi di base, calcolo della matrice inversa

Nel seguente tabella vengono assegnati uno spazio vettoriale e due basi $\mathcal{B}$ e $\widehat{\mathcal{B}}$ dello spazio stesso (non sarebbe male verificare che si tratti effettivamente di basi), oltre alla solita base canonica $\mathcal{C}$. Determinare le matrici di cambio di base tra le basi indicate nelle restanti colonne.

	Spazio	$\mathcal{B}$	$\widehat{\mathcal{B}}$	$\mathcal{C} \to \mathcal{B}$	$\mathcal{C} \to \widehat{\mathcal{B}}$	$\widehat{\mathcal{B}} \to \mathcal{B}$
1)	$\mathbb{R}^2$	$v_1 = (2,3)$ $v_2 = (1,5)$	$w_1 = (-3,4)$ $w_2 = (1,-3)$	$\frac{1}{7}\begin{pmatrix} 5 & -1 \\ -3 & 2 \end{pmatrix}$	$\frac{1}{5}\begin{pmatrix} -3 & -1 \\ -4 & -3 \end{pmatrix}$	$\frac{1}{7}\begin{pmatrix} -19 & 8 \\ 17 & -9 \end{pmatrix}$
2)	$\mathbb{R}^2$	$v_1 = (-1,4)$ $v_2 = (2,5)$	$w_1 = (1,6)$ $w_2 = (1,7)$	$-\frac{1}{13}\begin{pmatrix} 5 & -2 \\ -4 & -1 \end{pmatrix}$	$\begin{pmatrix} 7 & -1 \\ -6 & 1 \end{pmatrix}$	$-\frac{1}{13}\begin{pmatrix} -7 & -9 \\ -10 & -11 \end{pmatrix}$
3)	$\mathbb{R}^2$	$v_1 = (0,1)$ $v_2 = (1,0)$	$w_1 = (1,3)$ $w_2 = (2,1)$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	$-\frac{1}{5}\begin{pmatrix} 1 & -2 \\ -3 & 1 \end{pmatrix}$	$\begin{pmatrix} 3 & 1 \\ 1 & 2 \end{pmatrix}$
4)	$\mathbb{R}^3$	$v_1 = (1,2,1)$ $v_2 = (2,3,0)$ $v_3 = (0,1,1)$	$w_1 = (1,2,0)$ $w_2 = (0,-1,1)$ $w_3 = (3,1,4)$	$\begin{pmatrix} 3 & -2 & 2 \\ -1 & 1 & -1 \\ -3 & 2 & -1 \end{pmatrix}$	$\begin{pmatrix} -5 & 3 & 3 \\ -8 & 4 & 5 \\ 2 & -1 & -1 \end{pmatrix}$	$\begin{pmatrix} -1 & 4 & 15 \\ 1 & -2 & -6 \\ 1 & -3 & -11 \end{pmatrix}$
5)	$\mathbb{R}^3$	$v_1 = (0,1,1)$ $v_2 = (-1,1,0)$ $v_3 = (2,0,1)$	$w_1 = (1,1,1)$ $w_2 = (2,0,1)$ $w_3 = (2,-3,0)$	$\begin{pmatrix} -1 & -1 & 2 \\ 1 & 2 & -2 \\ 1 & 1 & -1 \end{pmatrix}$	$\begin{pmatrix} -3 & -2 & 6 \\ 3 & 2 & -5 \\ -1 & -1 & 2 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & -4 \\ 1 & 1 & -1 \end{pmatrix}$
6)	$\mathbb{R}^4$	$v_1 = (1,0,1,0)$ $v_2 = (2,1,1,3)$ $v_3 = (1,2,0,1)$ $v_4 = (1,2,0,0)$	$w_1 = (0,1,-1,2)$ $w_2 = (0,2,1,1)$ $w_3 = (1,0,1,-1)$ $w_4 = (3,1,1,1)$	$\begin{pmatrix} -2 & 1 & 3 & 0 \\ 2 & -1 & -2 & 0 \\ -6 & 3 & 6 & 1 \\ 5 & -2 & -5 & -1 \end{pmatrix}$	$\frac{1}{3}\begin{pmatrix} 2 & 6 & -7 & 5 \\ -1 & 0 & 2 & 1 \\ 3 & 9 & -9 & -9 \\ 0 & -3 & 3 & 3 \end{pmatrix}$	$\begin{pmatrix} -2 & 5 & 1 & -2 \\ 1 & -4 & 0 & 3 \\ -1 & 13 & -1 & -8 \\ 1 & -10 & 1 & 7 \end{pmatrix}$
7)	$\mathbb{R}_{\leq 2}[x]$	$v_1 = x^2$ $v_2 = x^2 + x$ $v_3 = x^2 + x + 1$	$w_1 = x^2 + x$ $w_2 = x^2 + x + 1$ $w_3 = x$	$\begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 & -1 \\ 0 & 0 & 1 \\ -1 & 1 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & -1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$
8)	$\mathbb{R}_{\leq 3}[x]$	$v_1 = x + 1$ $v_2 = x^3 + x$ $v_3 = x^3 - x - 1$ $v_4 = -x^2 + 10x + 2$	$w_1 = x^3 + x^2 + x$ $w_2 = x^3 + x^2 + 1$ $w_3 = x^3 + x + 1$ $w_4 = x^2 + x + 1$	$\begin{pmatrix} 1 & -6 & -1 & 2 \\ 0 & 8 & 1 & -1 \\ 1 & -8 & -1 & 1 \\ 0 & -1 & 0 & 0 \end{pmatrix}$	$\frac{1}{3}\begin{pmatrix} 1 & 1 & 1 & -2 \\ 1 & 1 & -2 & 1 \\ 1 & -2 & 1 & 1 \\ -2 & 1 & 1 & 1 \end{pmatrix}$	$\begin{pmatrix} -6 & -3 & 2 & -5 \\ 9 & 7 & 0 & 8 \\ -8 & -6 & 1 & -8 \\ -1 & -1 & 0 & -1 \end{pmatrix}$

$C \rightsquigarrow B$

$$\begin{pmatrix} 1 & 0 & 0 & -1 \\ 1 & 0 & -1 & 0 \\ -1 & 0 & 1 & 1 \\ -2 & 1 & 1 & 1 \end{pmatrix}$$

Stesse domande dell'esercizio precedente, ma nello spazio $M_{2\times 2}$ utilizzando le basi seguenti:

$$v_1 = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}, \quad v_2 = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}, \quad v_3 = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}, \quad v_4 = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}.$$

$$w_1 = \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}, \quad w_2 = \begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix}, \quad w_3 = \begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix}, \quad w_4 = \begin{pmatrix} 1 & 0 \\ -1 & 0 \end{pmatrix}.$$

$$\frac{1}{2}\begin{pmatrix} 1 & 1 & 1 & -1 \\ -1 & 1 & -1 & 1 \\ 1 & -1 & 1 & 1 \\ 1 & -1 & -1 & 1 \end{pmatrix}$$

$C \rightsquigarrow \widehat{B}$

$\widehat{B} \rightsquigarrow B$

$$\begin{pmatrix} 1 & -1 & -1 & 1 \\ 1 & 0 & -1 & 2 \\ -1 & 1 & 2 & -2 \\ -1 & 2 & 2 & -3 \end{pmatrix}$$