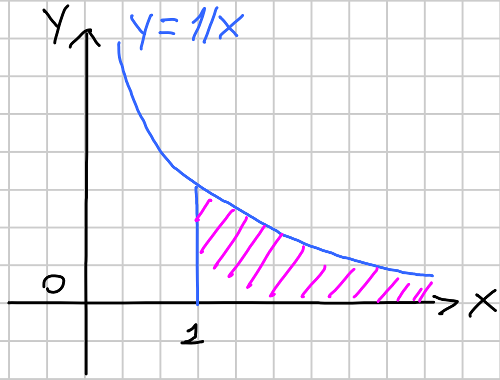
BY GIMUSI 17/10/2015

stabilire se le funzioni x^2+y^2-xy e e^x-y, ammettono limite per x^2+y^2-->+inf nel dominio D1=((x,y) app. ad R^2: x=>1, 0<=y<=1/x) ed al dominio D2=((x,y)app. ad R^2: x^1/2<=y<=x).

$$\begin{cases} f(x,y) = x^2 + y^2 - xy \\ g(x,y) = e^{x-y} \end{cases} \qquad \begin{cases} D_1: \begin{cases} x \geq 1 \\ 0 \leq y \leq 1/x \end{cases} \\ D_2: \sqrt{x} \leq y \leq x \end{cases}$$

DOMINIO D_1: $\begin{cases} x \geq 1 \\ 0 \leq y \leq 1/x \end{cases}$



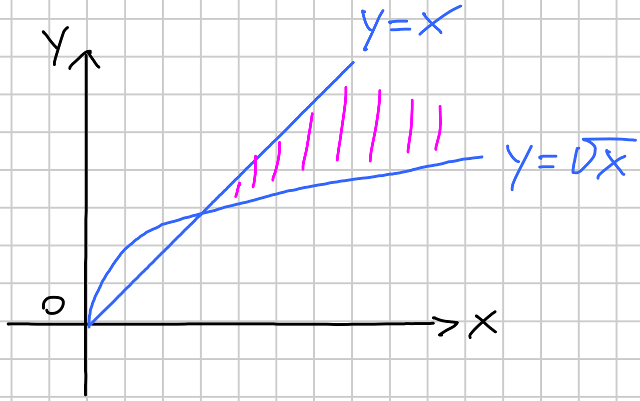
$$\lim_{x^2+y^2 \to +\infty} x^2 + y^2 - xy = \lim_{\rho \to +\infty} \rho^2 - \rho^2 \sin\theta \cos\theta =$$

$$= \lim_{\rho \to +\infty} \overset{\to +\infty}{\rho^2} \left(\overset{\to 1}{1 - \frac{1}{2} \sin 2\theta} \right) = +\infty$$

$$\lim_{x^2+y^2 \to +\infty} e^{x-y} = +\infty$$

$$e^{\overset{\to +\infty}{x - 1/x}} \leq e^{x-y} \leq e^{\overset{\to +\infty}{x}}$$

DOMINIO $D_2: \sqrt{x} \le y \le x$



$$\lim_{x^2+y^2 \to +\infty} x^2+y^2-xy = \lim_{\rho \to +\infty} \rho^2 - \rho^2 \sin\theta\cos\theta =$$

$$= \lim_{\rho \to +\infty} \rho^2 \left(1 - \frac{1}{2}\sin 2\theta\right) = +\infty$$

$$\underbrace{\rho^2\left(1-\frac{1}{2}\frac{\sqrt{2}}{2}\right)}_{\to +\infty} \le \underbrace{\rho^2\left(1-\frac{1}{2}\sin 2\theta\right)}_{\to +\infty} \le \underbrace{\rho^2}_{\to +\infty}$$

$$\lim_{x^2+y^2 \to +\infty} e^{x-y} = N.E.$$

$$\lim_{\substack{x^2+y^2 \to +\infty \\ y=x}} e^0 = 1 \ne \lim_{\substack{x^2+y^2 \to +\infty \\ y=\sqrt{x}}} e^{x-\sqrt{x}} = +\infty$$