BY GIMUSI 16/02/13

Trovare il polinomio di Mac Laurin di ordine 11 della funzione
F(x)= (e^(-2x^2) - cos 2x) (sin (x^3) - (sin x)^3)

$$F(x)=\left(e^{-2x^2}-\cos(2x)\right)\left(\sin(x^3)-\sin^3 x\right)$$

$$\begin{cases} e^{-2x^2}=1-2x^2+\frac{1}{2}(-2x^2)^2+\frac{1}{6}(-2x^2)^3+\frac{1}{24}(-2x^2)^4+\frac{1}{120}(-2x^2)^5+o(x^{11}) \\ \cos(2x)=1-\frac{(2x)^2}{2}+\frac{1}{24}(2x)^4-\frac{1}{720}(2x)^6+\frac{1}{8!}(2x)^8-\frac{1}{10!}(2x)^{10}+o(x^{11}) \end{cases}$$

$$e^{-2x^2}-\cos(2x)=\left(2-\frac{2}{3}\right)x^4+\left(-\frac{4}{3}+\frac{4}{45}\right)x^6+\left(\frac{2}{3}-\frac{2^8}{8!}\right)x^8+o(x^8)$$

$\frac{-60+4}{45}$

$$\begin{cases} \sin(x^3)=x^3-\frac{1}{6}x^9+o(x^9) \\ \sin^3 x=\left(x-\frac{1}{6}x^3+\frac{1}{120}x^5+o(x^5)\right)^3= \end{cases}$$

$$=x^3+3\left(-\frac{1}{6}\right)x^5+3\left(\frac{1}{36}\right)x^7+3\left(\frac{1}{120}\right)x^7+o(x^7)$$

$\frac{10+3}{120}$

$$\sin(x^3)-\sin^3 x=\frac{x^5}{2}-\left(\frac{1}{12}+\frac{1}{40}\right)x^7+o(x^7)$$

$$F(x)=\left(e^{-2x^2}-\cos(2x)\right)\left(\sin(x^3)-\sin^3 x\right)=$$

$$=\left(\frac{4}{3}x^4-\frac{56}{45}x^6+o(x^7)\right)\left(\frac{x^5}{2}-\frac{13}{120}x^7+o(x^7)\right)=$$

$\frac{56+13}{90}=\frac{69}{90}$

$$=\frac{2}{3}x^9-\frac{28}{45}x^{11}-\frac{13}{90}x^{11}+o(x^{11})=$$

$$=\frac{2}{3}x^9-\frac{23}{30}x^{11}+o(x^{11})$$