$$\lim_{n\to+\infty} \frac{(n!)^2}{(n^2)!} = 0$$

APPLICO IL CRITERIO DEL RAPPORTO PER LE SUCCESSIONI

$$\lim_{n\to+\infty} \frac{a_{n+1}}{a_n} \underset{=}{\overset{\text{OK}}{=}} \frac{(n+1)!^2}{((n+1)^2)!} \cdot \frac{(n^2)!}{(n!)^2} =$$

$$= \lim_{n\to+\infty} \frac{(n+1)^2 \cdot \cancel{(n!)^2}}{(n^2+2n+1)!} \cdot \frac{(n^2)!}{\cancel{(n!)^2}} = \text{OK}$$

$$= \lim_{n\to+\infty} \frac{\cancel{(n+1)^2} \cdot (n^2)!}{\underbrace{\cancel{(n^2+2n+1)}}_{(n+1)^2} (n^2+2n)!} \overset{\text{OK}}{=} \lim_{n\to+\infty} \frac{(n^2)!}{(n^2+2n)((n-1)^2)!} =$$

NO!!! LIMITE FATTO METÀ PER VOLTA!!!

$$= \lim_{n\to+\infty} \frac{(n^2)!}{(n^2+2n)\left(n^2\underbrace{\left(1-\frac{1}{n}\right)^2}_{\downarrow\ 1}\right)!} =$$

$$= \lim_{n\to+\infty} \frac{(n^2)!}{(n^2+2n)(n^2)!} = 0$$

Quindi a_n è strettamente decrescente ed avrà come limite $\ell = 0$

$$\lim_{n\to+\infty} \frac{(n!)^2}{(n^2)!} \to 0$$

$$\frac{a_{n+1}}{a_n} = \frac{[(n+1)!]^2}{[(n+1)^2]!} \frac{(n^2)!}{(n!)^2} = \left[\frac{(n+1)!}{n!}\right]^2 \frac{(n^2)!}{(n^2+2n+1)!} =$$

$$= \left[\frac{(n+1)\,\cancel{n!}}{\cancel{n!}}\right]^2 \frac{(n^2)!}{\cancel{(n^2+2n+1)}(n^2+2n)!} = \frac{(n^2)!}{(n^2+2n)!} =$$

$$= \frac{\cancel{(n^2)!}}{(n^2+2n)(n^2+2n-1)\dots(n^2+1)\cancel{(n^2)!}} \to 0$$