BY GIMUSI 29/01/2015

$$\lim_{n\to+\infty}\frac{n^{n+1}+2^{n^2}}{(2n)!}\to+\infty$$

$$\frac{n^{n+1}+2^{n^2}}{(2n)!}\geq\frac{2^{n^2}}{(2n)!}\geq\frac{2^{n^2}}{(2n)^{2n}}=\Bigg(\underbrace{\frac{2^n}{(2n)^2}}_{\to+\infty}\Bigg)^n\to+\infty$$

$$\frac{2^n}{(2n)^2}=\frac{1}{4}\,\frac{2^n}{n^2}\to+\infty$$

$$2^n\geq n^2\qquad\forall n\geq 4$$

DIM

$n=0\qquad 2^0=1\geq 0\qquad$ (PASSO BASE)

$2^n\geq n^2\Rightarrow 2^{n+1}\geq(n+1)^2\qquad$ (PASSO INDUTT.)

$$2^{n+1}=2\cdot 2^n\geq 2\cdot n^2\overset{?}{\geq}(n+1)^2$$

$$2\cdot n^2\geq(n+1)^2=n^2+2n+1$$

$$2n^2\geq n^2+2n+1\Rightarrow n^2-2n-1\geq 0$$

$$n=\frac{2\pm\sqrt{8}}{2}\leadsto n>1+\sqrt{2}\leadsto n\geq 3$$

$n=3\qquad 2^3=8\overset{?}{\geq}3^2=9\qquad$ NO!

$n=4\qquad 2^4=16\geq 4^2=16\qquad$ OK!