BY GIMUSI 13/01/15

Limiti 8

Argomenti: esistenza e non esistenza di limiti **Difficoltà:** ⋆⋆⋆

Prerequisiti: uso di successioni e sottosuccessioni per la non esistenza di limiti

In ogni riga della seguente tabella è indicata una successione e quattro sue sottosuccessioni (descritte mediante gli indici n_k che ne fanno parte). Si chiede di determinare, ovviamente quando esiste, il limite delle sottosuccessioni.

	Successione	n_k	Limite	n_k	Limite	n_k	Limite	n_k	Limite
1)	$(-1)^n$	$2k$	1	$2k+1$	−1	k^2	—	$k!$	1
2)	$3+(-1)^n$	$3k+2$	—	$22k+1$	2	k^2+k	4	22^k	4
3)	$(-n)^n$	$2k$	+∞	$2k+1$	−∞	k^2	—	3^k	−∞
4)	$\sin(n\pi/2)$	$2k$	0	$2k+1$	—	$4k+1$	−1	$8k-3$	1
5)	$\cos(n\pi/6)$	$6k$	—	$12k$	1	$12k+3$	0	$2k+1$	—

Calcolare i limiti delle seguenti successioni.

	Successione a)	Limite	Successione b)	Limite	Successione c)	Limite
6)	$n^8+(-1)^n n^5$	+∞	$n^5+(-1)^n n^8$	—	$n^5+(-\sqrt{n})^n$	—
7)	$2^{\sin(\pi n)}$	1	$3^{5+\cos(\pi n)}$	—	$(5+\cos(\pi n))^3$	—
8)	$(n-n^2)^n$	—	$(n^2-n)^n$	+∞	$(n-n^2)^{2n+1}$	+∞
9)	$n^2-\cos n^3$	+∞	$\left(3+\cos\left(\frac{\pi}{22}n\right)\right)^n$	+∞	$\left(2+\cos\left(\frac{\pi}{22}n\right)\right)^n$	—

Calcolare i seguenti limiti di funzione.

	Funzione a)	Limite	Funzione b)	Limite
10)	$\lim\limits_{x\to+\infty}\sin x$	—	$\lim\limits_{x\to-\infty}\cos x^2$	—
11)	$\lim\limits_{x\to0^+}\sin\frac{1}{x}$	—	$\lim\limits_{x\to+\infty}\cos\frac{1}{\log x}$	1
12)	$\lim\limits_{x\to+\infty}\sin^2(\log x+2)$	—	$\lim\limits_{x\to-\infty}\log^2(\sin x+2)$	—
13)	$\lim\limits_{x\to+\infty}\cos x+\cos\frac{1}{x}$	—	$\lim\limits_{x\to0^-}\cos x+\cos\frac{1}{x}$	—
14)	$\lim\limits_{x\to0^-}\cos x\cdot\cos\frac{1}{x}$	—	$\lim\limits_{x\to+\infty}\sin x\cdot\sin\frac{1}{x}$	0

1/5

In ogni riga della seguente tabella è indicata una successione e quattro sue sottosuccessioni (descritte mediante gli indici n_k che ne fanno parte). Si chiede di determinare, ovviamente quando esiste, il limite delle sottosuccessioni.

	Successione	n_k	Limite	n_k	Limite	n_k	Limite	n_k	Limite
1)	$(-1)^n$	$2k$	1	$2k+1$	-1	k^2	—	$k!$	1

$$(-1)^{2n} = (1)^n \to 1 \qquad (-1)^{2n+1} = -1(-1)^{2n} \to -1$$

$$(-1)^{n^2} \to \begin{cases} 1 & n^2 \text{ PARI} \\ -1 & n^2 \text{ DISPARI} \end{cases} \qquad (-1)^{n!} = \left[(-1)^2\right]^{n!/2} \to 1$$

2)	$3+(-1)^n$	$3k+2$	—	$22k+1$	2	k^2+k	4	22^k	4

$$3+(-1)^{3n+2} = 3+(-1)^2(-1)^{3n} = 3+(-1)^n \quad \text{N.E.}$$

$$3+(-1)^{22n+1} = 3+(-1)(-1)^{22n} = 3-(1)^n \to 2$$

$$3+(-1)^{n^2+n} = 3+\left[(-1)^k\right]^{n+1} \to 4$$

$$3+(-1)^{22n} = 3+\left[(-1)^{22}\right]^n \to 4$$

3)	$(-n)^n$	$2k$	$+\infty$	$2k+1$	$-\infty$	k^2	—	3^k	$-\infty$

$$(-2n)^{2n} = (4n^2)^n \to +\infty$$

$$(-2n-1)^{2n+1} = (-2n-1)\left[(-2n-1)^2\right]^n \to -\infty$$

$$(-n^2)^{n^2} \to \begin{cases} +\infty & n^2 \text{ PARI} \\ -\infty & n^2 \text{ DISPARI} \end{cases} \qquad (-3^n)^{(3^n)} \to -\infty$$

4)	$\sin(n\pi/2)$	$2k$	0	$2k+1$	—	$4k+1$	-1	$8k-3$	1

$$\sin(2n\pi/2) = \sin(n\pi) \to 0$$

$$\sin((2n+1)(\pi/2)) = \sin(n\pi+\pi/2) = -\cos(n\pi) \to \begin{cases} +1 & K \text{ DISPARI} \\ -1 & K \text{ PARI} \end{cases}$$

$$\sin(2n\pi+\pi/2) = -\cos(2n\pi) \to -1$$

$$\sin(4n\pi-3\pi/2) = -\sin(3\pi/2-4n\pi) = \cos(4n\pi) \to 1$$

5)

$\cos(n\pi/6)$	$6k$	—	$12k$	1	$12k+3$	0	$2k+1$	—

$$\cos(n\pi) \to \begin{cases} -1 & K \text{ DISPARI} \\ +1 & K \text{ PARI} \end{cases} \qquad \cos(2n\pi) \to 1$$

$$\cos(2n\pi + \pi/2) = -\sin(2n\pi) \to 0 \qquad \cos(n\pi/3 + \pi/6) \text{ N.E.}$$

6.a) $$n^8 + (-1)^n \cdot n^5 = n^8\left(1 + \overset{\to 1}{\frac{(-1)^n}{n^3}}\right) \to +\infty$$

6.b) $$n^5 + (-1)^n n^8 = n^8\left(\frac{1}{n^3} + (-1)^n\right) \to \begin{cases} +\infty & n = 2k \\ -\infty & n = 2k+1 \end{cases} \qquad \text{N.E.}$$

6.c) $$n^5 + (-\sqrt{n})^n = n^5 + (-1)^n \cdot n^{n/2} = n^{n/2}\left(\frac{1}{n^{\frac{n}{2}-5}} + (-1)^n\right) \to \begin{cases} +\infty & n = 2k \\ -\infty & n = 2k+1 \end{cases} \qquad \text{N.E.}$$

7.a) $$2^{\sin(\pi n)} = 2^0 = 1$$

7.b) $$3^{5+\cos(\pi n)} = \begin{cases} 3^6 & n = 2k \\ 3^4 & n = 2k+1 \end{cases} \qquad \text{N.E.}$$

7.c) $$(5 + \cos(\pi n))^3 = \begin{cases} 6^3 & n = 2k \\ 4^3 & n = 2k+1 \end{cases} \qquad \text{N.E.}$$

8.a) $$(n - n^2)^n = \sqrt[n]{n^2}\left(\frac{1}{n} - 1\right)^n = \overset{\to +\infty}{\sqrt[n]{n^2}}\,(-1)^n \overset{\to 1/e}{\left(1 - \frac{1}{n}\right)^n} \to$$

$$\to \begin{cases} +\infty & n = 2k \\ -\infty & n = 2k+1 \end{cases} \qquad \text{N.E.}$$

8.b) $$(n^2 - n)^n = \overset{\to +\infty}{\sqrt[n]{n^2}} \overset{\to 1/e}{\left(1 - \frac{1}{n}\right)^n} \to +\infty$$

8.c) $$(n - n^2)^{2n+1} = (-1)^{2n+1} (n^2 - n)\,(n^2 - n)^{2n} =$$

$$= \overset{\to 1}{(-1)^{2n+1}} \overset{\to +\infty}{(n^2 - n)} \overset{\to +\infty}{\sqrt[2n]{n}} \overset{\to 1/e^2}{\left[\left(1 - \frac{1}{n}\right)^n\right]^2} \to +\infty$$

9.a) $n^2 - \cos n^3 = n^2\left(1 - \frac{\cos n^3}{n^2}\right) \to +\infty$

9.b) $\left(3 + \cos\left(\frac{\pi}{22} n\right)\right)^n \geq 2^n \to +\infty$

9.c) $\left(2 + \cos\left(\frac{\pi}{22} n\right)\right)^n \to \begin{cases} +\infty & \frac{n}{22} \neq k\frac{\pi}{2} \\ 1 & \frac{n}{22} = k\frac{\pi}{2} \end{cases}$ N.E.

10.a) $\lim_{x \to +\infty} \sin x$ N.E.

$$\begin{cases} a_n = 2\pi n \to +\infty & \sin(a_n) \to 0 \\ b_n = \frac{\pi}{2} + 2\pi n \to +\infty & \sin(b_n) \to 1 \end{cases}$$

10.b) $\lim_{x \to -\infty} \cos x^2$ N.E.

$$\begin{cases} a_n = -\sqrt{2\pi n} \to -\infty & \cos(a_n^2) = \cos(2\pi n) \to 1 \\ b_n = -\sqrt{\frac{\pi}{2} + 2\pi n} \to -\infty & \cos(b_n^2) = \cos\left(\frac{\pi}{2} + 2\pi n\right) \to 0 \end{cases}$$

11.a) $\lim_{x \to 0^+} \sin\frac{1}{x} = \lim_{y \to +\infty} \sin(y)$ N.E.

11.b) $\lim_{x \to +\infty} \cos \underbrace{\frac{1}{\log x}}_{\to 0^+} = \lim_{y \to 0^+} \cos y \to 1$

12.a) $\lim_{x \to +\infty} \sin^2(\underbrace{\log x + 2}_{\to +\infty}) = \lim_{y \to +\infty} \sin^2(y)$ N.E.

12.b) $\lim_{x \to -\infty} \log^2(\sin x + 2)$ N.E.

$$\begin{cases} a_n = -2\pi n \to -\infty & \log^2(\sin(a_n) + 2) = \log^2(2) > 0 \\ b_n = -\frac{\pi}{2} - 2\pi n \to -\infty & \log^2(\sin(b_n) + 2) = \log^2(1) = 0 \end{cases}$$

13.a) $\lim_{x \to +\infty} \cos x + \cos \frac{1}{x}$ N.E.

$$\begin{cases} a_n = 2\pi n \to +\infty & \cos(2\pi n) + \cos\left(\frac{1}{2\pi n}\right) \to 2 \\ b_n = \frac{\pi}{2} + 2\pi n \to +\infty & \cos\left(\frac{\pi}{2} + 2\pi n\right) + \cos\left(\frac{1}{\pi/2 + 2\pi n}\right) \to 1 \end{cases}$$

13.b) $\lim_{x \to 0^-} \cos x + \cos \frac{1}{x} = \lim_{y \to 0^+} \cos(-y) + \cos\left(\frac{-1}{y}\right) = \lim_{y \to 0^+} \cos y + \cos \frac{1}{y}$ N.E.

15.a) $\lim_{x \to 0^-} \overset{\to 1}{\cos x} \cdot \overset{\text{N.E.}}{\cos \frac{1}{x}}$ N.E.

15.b) $\lim_{x \to +\infty} \sin x \cdot \sin \frac{1}{x} \to 0$

$$\overset{\to 0}{-\sin \frac{1}{x}} \le \overset{\to 0}{\sin x \cdot \sin \frac{1}{x}} \le \overset{\to 0}{\sin \frac{1}{x}}$$