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Limiti 7

Argomenti: limiti di funzioni e successioni **Difficoltà:** $\star\star\star$

Prerequisiti: limiti notevoli, criterio funzioni $\to$ successioni

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Calcolare i limiti delle seguenti funzioni (tutti i limiti si intendono per $x \to 0$).

	a) Funzione	Limite	b) Funzione	Limite
1)	$\dfrac{\sin(2x+x^3)+x}{\arctan(3x-x^5)-x}$	$3/2$	$\dfrac{\cos(x+x^2)-1}{x\sin x + x\log(1+2x)}$	$-1/6$
2)	$\dfrac{e^{x+\sin(2x)}-1}{\arcsin(5x)}$	$3/5$	$\dfrac{5^{x+\sin(2x)}-\cos x}{\arcsin(5x)}$	$3\log 5/5$
3)	$\dfrac{e^{2x}+\cos(3x)-2}{\arctan x + \arcsin x}$	1	$\dfrac{e^{2x^2}+\cos(3x)-2}{\arctan x^2 + \arcsin x}$	0
4)	$\dfrac{\log(\cos x)}{x^2}$	$-1/2$	$(\cos x)^{1/\sin^2 x}$	$e^{-1/2}$
5)	$(1+\sin(2x^2))^{1/\arctan^2 x}$	e^2	$(1-\sin^2 x)^{1/\tan x}$	1
6)	$(2-\cos x)^{x/\sin x^3}$	$\sqrt{e}$	$(2-\cos^{30} x)^{x^3/\sin x}$	1
7)	$\dfrac{1-\cos^2 x^3}{1-\cos^3 x^2}$	0	$\dfrac{1-\cos^2 x^3}{1-\cos^3 x^2}\cdot\cos\dfrac{1}{x}$	0

Calcolare i limiti delle seguenti successioni.

	a) Successione	Limite	b) Successione	Limite
8)	$\left(\log(2+n^2)-2\log n\right)n^2$	2	$\left(2-\cos\dfrac{3}{n+n^2}\right)^{n^4}$	$e^{3/2}$
9)	$n\left(\sqrt[n]{n}-1\right)$	$+\infty$	$\dfrac{n}{\log n}\left(\sqrt[n]{2n}-1\right)$	1
10)	$\left(\cos\dfrac{1}{n+2}-\sqrt[n]{2}\right)^n$	0	$\left(n+\sqrt{n}+\sqrt[3]{n}\right)^{1/\log n}$	e
11)	$\left(2+\dfrac{1}{n}\right)^n-2^n$	$+\infty$	$\left(2+\dfrac{1}{n\cdot 2^n}\right)^n-2^n$	$1/2$

Si raccomanda di svolgere questa scheda di esercizi almeno due volte: la prima subito dopo aver visto i limiti notevoli, esplicitando tutti i dettagli; la seconda dopo aver capito come si usano gli sviluppini, scrivendo solo lo stretto necessario in tempi rapidi.

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1.a) $\dfrac{\sin(2x+x^3)+x}{\arctan(3x-x^5)-x} = \dfrac{2x+x^3+x+o(x)}{3x-x^5+o(x)-x} = \dfrac{3x+o(x)}{2x+o(x)} \to \dfrac{3}{2}$

1.b) $\dfrac{\cos(x+x^2)-1}{x\sin x + x\log(1+2x)} = \dfrac{\cancel{1}-(x+x^2)^2/2+o(x^2)-\cancel{1}}{x^2+o(x^2)+2x^2+o(x^2)} =$

$= \dfrac{-x^2/2+o(x^2)}{3x^2+o(x^2)} \to -1/6$

2.a) $\dfrac{e^{x+\sin 2x}-1}{\arcsin(5x)} = \dfrac{e^{x+2x+o(x)}-1}{5x+o(x)} = \dfrac{\cancel{1}+3x+o(x)-\cancel{1}}{5x+o(x)} \to 3/5$

2.b) $\dfrac{5^{x+\sin 2x}-\cos x}{\arcsin(5x)} = \dfrac{5^{3x+o(x)}-1+o(x)}{5x+o(x)} = \dfrac{1+3\log 5\cdot x+o(x)-1}{5x+o(x)} \to \dfrac{3\log 5}{5}$

3.a) $\dfrac{e^{2x}+\cos(3x)-2}{\arctan x+\arcsin x} = \dfrac{\cancel{1}+2x+o(x)+\cancel{1}+o(x)-\cancel{2}}{x+o(x)+x+o(x)} = \dfrac{2x+o(x)}{2x+o(x)} \to 1$

3.b) $\dfrac{e^{2x^2}+\cos(3x)-2}{\arctan x^2+\arcsin x} = \dfrac{1+o(x)+1+o(x)-2}{x^2+o(x^2)+x+o(x)} = \dfrac{o(x)}{x+o(x)} \to 0$

4.a) $\dfrac{\log(\cos x)}{x^2} = \dfrac{\log(1-x^2/2+o(x^2))}{x^2} = \dfrac{-x^2/2+o(x^2)}{x^2} = -1/2$

4.b) $(\cos x)^{1/\sin^2 x} = e^{\frac{\log(\cos x)}{\sin^2 x}} = e^{\frac{-x^2/2+o(x^2)}{x^2+o(x^2)}} \to e^{-1/2}$

5.a) $(1+\sin(2x^2))^{1/\arctan^2 x} = e^{\frac{\log(1+\sin(2x^2))}{\arctan^2 x}} = e^{\frac{2x^2+o(x^2)}{x^2+o(x^2)}} \to e^2$

5.b) $(1-\sin^2 x)^{1/\tan x} = e^{\frac{\log(1-\sin^2 x)}{\tan x}} = e^{\frac{x^2+o(x^2)}{x+o(x)}} \to 1$

6.a) $(2-\cos x)^{x/\sin x^3} = e^{\frac{x\log(2-\cos x)}{\sin x^3}} = e^{\frac{x^3/2+o(x^3)}{x^3+o(x^3)}} \to \sqrt{e}$

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6.b) $(2-\cos^{30}x)^{x^3/\sin x} \to 1 \qquad (1^0)$

7.a) $\dfrac{1-\cos^2 x^3}{1-\cos^3 x^2} = \dfrac{1-(1-x^6/2+o(x^6))^2}{1-(1-x^5/2+o(x^5))^3} = \dfrac{\cancel{1}-\cancel{1}+x^6+o(x^{12})}{\cancel{1}-\cancel{1}+x^5+o(x^8)} \to 0$

7.b) $\dfrac{\overset{\to 0}{1-\cos^2 x^3}}{1-\cos^3 x^2}\;\overset{\ge -1,\ \le 1}{\cos\dfrac{1}{x}} \to 0$

8.a) $(\log(2+m^2)-2\log m)\,m^2 = \log\left(\overset{\to e^2}{\dfrac{2+m^2}{m^2}}\right)^{m^2} \to 2$

8.b) $\left(2-\cos\dfrac{3}{m+m^2}\right)^{m^5} = \left(2-1+\dfrac{1}{2}\left(\dfrac{3}{m+m^2}\right)^2+o\left(\dfrac{1}{m^5}\right)\right)^{m^5} =$

$$= e^{m^5\log\left(1+\frac{1}{2}\left(\frac{3}{m+m^2}\right)^2+o\left(\frac{1}{m^5}\right)\right)} =$$

$$= e^{\overset{\to 3/2}{m^5\cdot\frac{1}{2}\left(\frac{3}{m^2+m}\right)^2}+\overset{\to 0}{m^5 o\left(\frac{1}{m^5}\right)}} \to e^{3/2}$$

9.a) $m(\sqrt[m]{m}-1) = \dfrac{m^{1/m}-1}{1/m} = \dfrac{e^{\frac{\log m}{m}}-1}{\frac{\log m}{m}}\,\log m \to +\infty$

9.b) $\dfrac{m}{\log m}(\sqrt[m]{2m}-1) = \dfrac{1}{\log m}\,\dfrac{(2m)^{1/m}-1}{1/m} = \overset{\to 1}{\dfrac{\log 2m}{\log m}}\,\overset{\to 1}{\dfrac{e^{\frac{\log 2m}{m}}-1}{\frac{\log 2m}{m}}} \to 1$

10.a) $\left(\cos\dfrac{1}{m+2}-\sqrt[m]{2}\right)^m = \left(\cancel{1}-\cancel{1}-\dfrac{\log 2}{m}+o\left(\dfrac{1}{m}\right)\right)^m \to 0$

10.b) $(m+\sqrt{m}+\sqrt[3]{m})^{1/\log m} = e^{\overset{\to 1}{\frac{\log(m+\sqrt{m}+\sqrt[3]{m})}{\log m}}} \to e$

11.a) $\left(2+\frac{1}{n}\right)^n - 2^n = \underbrace{2^n}_{\to +\infty}\Big[\underbrace{\left(1+\frac{1}{2n}\right)^n - 1}_{\to \sqrt{e}-1}\Big] \to +\infty$

11.b) $\left(2+\frac{1}{n\cdot 2^n}\right)^n - 2^n = 2^n\left(1+\frac{1}{2n\cdot 2^n}\right)^n - 2^n$

$$\left(1+\frac{1}{2n\cdot 2^n}\right)^n = e^{n\log\left(1+\frac{1}{2n\cdot 2^n}\right)} = e^{\frac{1}{2^{n+1}}+o\left(\frac{1}{2^{n+1}}\right)} =$$

$$= 1+\frac{1}{2^{n+1}}+o\left(\frac{1}{2^{n+1}}\right)$$

$$2^n\left(1+\frac{1}{2n\cdot 2^n}\right)^n - 2^n = \cancel{2^n} + \frac{2^n}{2^{n+1}} - \cancel{2^n} + 2^n\, o\left(\frac{1}{2^{n+1}}\right) \to 1/2$$