BY GIMUSI 07/01/15

Serie 1

Argomenti: convergenza di serie a termini di segno costante **Difficoltà:** ⋆⋆

Prerequisiti: criterio del rapporto e della radice

Stabilire se le seguenti serie numeriche sono convergenti oppure no.

	a) Serie	Conv.?	b) Serie	Conv.?	c) Serie	Conv.?
1)	$\sum_{n=0}^{\infty} \frac{n^{33}}{3^n}$	SÌ	$\sum_{n=0}^{\infty} \frac{n^2+3}{n!}$	SÌ	$\sum_{n=0}^{\infty} \frac{4^n}{n!}$	SÌ
2)	$\sum_{n=0}^{\infty} \frac{3^n}{2^n}$	NO	$\sum_{n=0}^{\infty} \frac{n!}{3^{n^2}}$	SÌ	$\sum_{n=0}^{\infty} \frac{n(n+8)}{n!}$	SÌ
3)	$\sum_{n=0}^{\infty} \frac{3^n+4^n}{2^n+5^n}$	SÌ	$\sum_{n=0}^{\infty} \frac{n!-2^n}{55^n}$	NO	$\sum_{n=1}^{\infty} \left(\frac{n+3}{3n+1}\right)^{(n^2+1)/n}$	SÌ
4)	$\sum_{n=1}^{\infty} \left(1+\frac{1}{n}\right)^n$	NO	$\sum_{n=3}^{\infty} \left(1-\frac{2}{n}\right)^{n^2}$	SÌ	$\sum_{n=1}^{\infty} n^4 \left(1+\frac{3}{n}\right)^{-n^2}$	SÌ
5)	$\sum_{n=1}^{\infty} \binom{2n}{n}^{-1}$	NO	$\sum_{n=1}^{\infty} \frac{5^n \cdot n!}{2^n \cdot n^n}$	SÌ	$\sum_{n=71}^{\infty} \left(\sin \frac{n+1}{n-70}\right)^n$	SÌ
6)	$\sum_{n=1}^{\infty} \frac{2+n^n}{(2+n)^n}$	NO	$\sum_{n=1}^{\infty} \frac{n^{2n}}{(2n)!}$	SÌ	$\sum_{n=1}^{\infty} \sqrt[n]{\frac{n!+8}{8^n n!+n^8}}$	NO

Stabilire per quali valori del parametro $\alpha \in \mathbb{R}$ le seguenti serie numeriche convergono.

	a) Serie	α	b) Serie	α	c) Serie	α
7)	$\sum_{n=0}^{\infty} \frac{1}{\alpha^n}$	$(-\infty,-1) \cup (1,+\infty)$	$\sum_{n=0}^{\infty} (\alpha+1)^n$	$(-2,0)$	$\sum_{n=0}^{\infty} (2\alpha)^n$	$\left(-\frac{1}{2},\frac{1}{2}\right)$
8)	$\sum_{n=0}^{\infty} (\arctan \alpha)^n$	$\left(-\frac{\pi}{5},\frac{\pi}{5}\right)$	$\sum_{n=0}^{\infty} 3^n \alpha^{2n}$	$\left[0,\frac{\sqrt{3}}{3}\right)$	$\sum_{n=0}^{\infty} (\log \alpha)^n$	$\left(\frac{1}{e},e\right)$
9)	$\sum_{n=0}^{\infty} (\sin(\sin \alpha))^n$	$\mathbb{R}$	$\sum_{n=0}^{\infty} (\lvert\alpha\rvert-2)^n$	$(-3,-1) \cup (1,3)$	$\sum_{n=0}^{\infty} 3^{n^2} \lvert\alpha\rvert^n$	0
10)	$\sum_{n=0}^{\infty} \frac{1}{(n!)^\alpha}$	>0	$\sum_{n=0}^{\infty} \frac{2^n+n^4}{\lvert\alpha\rvert^n}$	$(-\infty,-2) \cup (2,+\infty)$	$\sum_{n=0}^{\infty} 3^n \lvert\alpha\rvert^{n^2}$	$(-1,1)$

1.a) $\sum_{n=0}^{\infty} \frac{n^{33}}{3^n}$ CONVERGE $\sqrt[n]{a_n} = \frac{\sqrt[n]{n^{33}}}{3} \to \frac{1}{3} < 1$

1.b) $\sum_{n=0}^{\infty} \frac{n^2+3}{n!}$ CONVERGE $\sqrt[n]{a_n} = \frac{\sqrt[n]{n^2+3}}{\sqrt[n]{n!}} \to 0 < 1$

1.c) $\sum_{n=0}^{\infty} \frac{5^n}{n!}$ CONVERGE $\sqrt[n]{a_n} = \frac{5}{\sqrt[n]{n!}} \to 0 < 1$

2.a) $\sum_{n=0}^{\infty} \frac{3^n}{2^n}$ NON CONVERGE $a_n \to +\infty$

2.b) $\sum_{n=0}^{\infty} \frac{n!}{3^{n^2}}$ CONVERGE $\sqrt[n]{a_n} = \frac{\sqrt[n]{n!}}{3^n} = \overset{\to 1/e}{\frac{\sqrt[n]{n!}}{n}} \cdot \overset{\to 0}{\frac{n}{3^n}} \to 0 < 1$

2.c) $\sum_{n=0}^{\infty} \frac{n(n+8)}{n!}$ CONVERGE $\sqrt[n]{a_n} = \frac{\sqrt[n]{n(n+8)}}{\sqrt[n]{n!}} \to 0 < 1$

3.a) $\sum_{n=0}^{\infty} \frac{3^n+5^n}{2^n+5^n}$ CONVERGE $\sqrt[n]{a_n} = \sqrt[n]{\frac{3^n+5^n}{2^n+5^n}} = \frac{5}{5}\sqrt[n]{\frac{(3/5)^n+1}{(2/5)^n+1}} \to \frac{5}{5} < 1$

3.b) $\sum_{n=0}^{\infty} \frac{n!-2^n}{55^n}$ NON CONVERGE $a_n = \frac{n!-2^n}{55^n} = \frac{n!}{55^n}(1-2^n/n!) \to +\infty$

3.c) $\sum_{n=1}^{\infty} \left(\frac{n+3}{3n+1}\right)^{(n^2+1)/n}$ CONVERGE $\sqrt[n]{a_n} = \left(\frac{n+3}{3n+1}\right)^{(n^2+1)/n^2} \to 1/3$

5.a) $\sum_{n=1}^{\infty} \left(1+\frac{1}{n}\right)^n$ NON CONVERGE $a_n \to e$

5.b) $\sum_{n=3}^{\infty} \left(1-\frac{2}{n}\right)^{n^2}$ CONVERGE $\sqrt[n]{a_n} = \left(1-\frac{2}{n}\right)^n \to 1/e^2 < 1$

5.c) $\sum_{n=1}^{\infty} n^5 \left(1+\frac{3}{n}\right)^{-n^2}$ CONVERGE $\sqrt[n]{a_n} = \overset{\to 1}{\sqrt[n]{n^5}} \overset{\to 1/e^3}{\left(1+\frac{3}{n}\right)^{-n}} \to 1/e^3 < 1$

5.a) $\sum_{n=1}^{\infty} \binom{2n}{n}^{-1}$ NON CONVERGE

$$\frac{a_{n+1}}{a_n} = \frac{(2n+2)!}{(n+1)!^2} \frac{n!^2}{2n!} = \frac{(2n+2)(2n+1)}{(n+1)^2} \to 4 > 1$$

5.b) $\sum_{n=1}^{\infty} \frac{5^n \cdot n!}{2^n \cdot n^n}$ CONVERGE $\sqrt[n]{a_n} = \frac{5}{2} \cdot \frac{\sqrt[n]{n!}}{n} \to \frac{5}{2e} < 1$

5.c) $\sum_{n=71}^{\infty} \left(\sin \frac{n+1}{n-70}\right)^n$ CONVERGE $\sqrt[n]{a_n} = \sin\overset{\to 1}{\left(\frac{n+1}{n-70}\right)} \to \sin(1) < 1$

6.a) $\sum_{n=1}^{\infty} \frac{2+n^n}{(2+n)^n}$ NON CONVERGE $a_n = \frac{2+n^n}{(2+n)^n} = \overset{\to 1}{\frac{n^n}{n^n}} \overset{\to 1/e^2}{\frac{\frac{2}{n^n}+1}{\left(\frac{2}{n}+1\right)^n}} \to \frac{1}{e^2}$

6.b) $\sum_{n=1}^{\infty} \frac{n^{2n}}{(2n)!}$ CONVERGE $\frac{a_{n+1}}{a_n} = \frac{n^{(2n+2)}}{(2n+2)!} \frac{(2n)!}{n^{2n}} = \frac{n^2}{(2n+2)(2n+1)} \to 1/4 < 1$

6.c) $\sum_{n=1}^{\infty} \sqrt[n]{\frac{n!+8}{8^n n! + n^8}}$ NON CONVERGE

$$a_n = \sqrt[n]{\frac{n!+8}{8^n n! + n^8}} = \frac{\sqrt[n]{n!}}{\sqrt[n]{n!}} \sqrt[n]{\frac{1+8/n!}{8^n + n^8/n!}} \to 1/8$$

7.a) $\sum_{n=0}^{\infty} \frac{1}{\alpha^n}$ CONVERGE PER $\alpha \in (-\infty, -1) \cup (1, +\infty)$

$a_n = \left(\frac{1}{\alpha}\right)^n$ SERIE GEOMETRICA $\to 1/\alpha \in (-1, 1)$

7.b) $\sum_{n=0}^{\infty} (\alpha+1)^n$ CONVERGE PER $\alpha \in (-2, 0)$

$a_n = (\alpha+1)^n$ SERIE GEOMETRICA $\to (\alpha+1) \in (-1, 1)$

7.c) $\sum_{n=0}^{\infty} (2\alpha)^n$ CONVERGE PER $\alpha \in (-1/2, 1/2)$

$a_n = (2\alpha)^n$ SERIE GEOMETRICA $\rightsquigarrow$ $(2\alpha) \in (-1,1)$

8.a) $\sum_{n=0}^{\infty} (\arctan\alpha)^n$ CONVERGE PER $\alpha \in (-\pi/4, \pi/4)$

$a_n = (\arctan\alpha)^n$ SERIE GEOMETRICA $\rightsquigarrow$ $\arctan\alpha \in (-1,1)$

8.b) $\sum_{n=0}^{\infty} 3^n \alpha^{2n}$ CONVERGE PER $\alpha \in [0, \sqrt{3}/3)$

$a_n = (3\alpha^2)^n$ SERIE GEOMETRICA $\rightsquigarrow$ $(3\alpha^2) \in (-1,1)$

8.c) $\sum_{n=0}^{\infty} (\log\alpha)^n$ CONVERGE PER $\alpha \in (1/e, e)$

$a_n = (\log\alpha)^n$ SERIE GEOMETRICA $\rightsquigarrow$ $\log\alpha \in (-1,1)$

9.a) $\sum_{n=0}^{\infty} (\sin(\sin\alpha))^n$ CONVERGE PER $\alpha \in \mathbb{R}$

$a_n = (\sin(\sin\alpha))^n$ SERIE GEOMETRICA $\rightsquigarrow$ $\sin(\sin\alpha) \in (-1,1)$

9.b) $\sum_{n=0}^{\infty} (|\alpha|-2)^n$ CONVERGE PER $\alpha \in (-3,-1) \cup (1,3)$

$a_n = (|\alpha|-2)^n$ SERIE GEOMETRICA $\rightsquigarrow$ $(|\alpha|-2) \in (-1,1)$

9.c) $\sum_{n=0}^{\infty} 3^{n^2} |\alpha|^n$ CONVERGE PER $\alpha = 0$

$\sqrt[n]{a_n} = 3^n |\alpha| \to 0 \iff \alpha = 0$ $\quad \sum 0$ CONVERGE

10.a) $\sum_{n=0}^{\infty} \frac{1}{(n!)^{\alpha}}$ CONVERGE PER $\alpha > 0$

$$\frac{a_{n+1}}{a_n} = \frac{(n!)^{\alpha}}{(n+1)!^{\alpha}} = \frac{1}{(n+1)^{\alpha}} < 1 \qquad (n+1)^{\alpha} > 1$$

$$\alpha \log(1+n) > 0 \qquad \alpha > 0$$

10.b) $\sum_{n=0}^{\infty} \frac{2^n + n^5}{|\alpha|^n}$ CONVERGE PER $\alpha \in (-\infty, -2) \cup (2, +\infty)$

$$\sqrt[n]{a_n} = \frac{\sqrt[n]{2^n + n^5}}{|\alpha|} \to \frac{2}{|\alpha|} < 1 \qquad |\alpha| > 2$$

10.c) $\sum_{n=0}^{\infty} 3^n |\alpha|^{n^2}$ CONVERGE PER $\alpha \in (-1, 1)$

$$\sqrt[n]{a_n} = 3|\alpha|^n \to \begin{cases} 0 & |\alpha| < 1 \rightsquigarrow \text{CONVERGE} \\ 3 & |\alpha| = 1 \\ +\infty & |\alpha| > 1 \end{cases} \quad \left.\begin{matrix} \\ \\ \end{matrix}\right\} \text{NON CONVERGE}$$