BY GIMUSI 11/07/15

$$\int_{\mathbb{R}^2} \frac{1}{7+(x^2+y^2)^\alpha}\,dx\,dy = \int_0^{2\pi}\int_0^{+\infty} \frac{\rho}{7+\rho^{2\alpha}}\,d\rho\,d\theta =$$

NON È IMPROPRIO

$$= 2\pi\int_0^{+\infty} \frac{\rho}{7+\rho^{2\alpha}}\,d\rho = \underbrace{2\pi\int_0^{1} \frac{\rho}{7+\rho^{2\alpha}}\,d\rho}_{\text{CONVERGE } \forall \alpha} + 2\pi\int_1^{+\infty} \frac{\rho}{7+\rho^{2\alpha}}\,d\rho$$

STUDIAMO $\displaystyle\int_1^{+\infty} \frac{\rho}{7+\rho^{2\alpha}}\,d\rho$ MEDIANTE CONFRONTO ASINTOTICO

SIANO $f(x) = \dfrac{x}{7+x^{2\alpha}}$ $\quad g(x) = \dfrac{1}{x^{2\alpha-1}}$

$$\lim_{x\to+\infty} \frac{f(x)}{g(x)} = \lim_{x\to+\infty} \frac{x^{2\alpha}}{7+x^{2\alpha}} = \lim_{x\to+\infty} \frac{1}{\frac{7}{x^{2\alpha}}+1} = \begin{cases} 1 & \alpha>0 \\ 1/8 & \alpha=0 \\ 0 & \alpha<0 \end{cases}$$

QUINDI PER $\alpha \geq 0$

$\displaystyle\int_1^{+\infty} \frac{\rho}{7+\rho^{2\alpha}}\,d\rho$ SI COMPORTA COME $\displaystyle\int_1^{+\infty} \frac{1}{\rho^{2\alpha-1}}\,d\rho$ E DUNQUE

$$\begin{cases} \text{CONVERGE PER } 2\alpha-1>1 & \rightsquigarrow \alpha>1 \\ \text{DIVERGE PER } 2\alpha-1\leq 1 & \rightsquigarrow 0\leq\alpha\leq 1 \end{cases}$$

PER $\alpha<0$, POSTO $\beta=-\alpha>0$

$$\int_1^{+\infty} \frac{\rho}{7+\rho^{2\alpha}}\,d\rho = \int_1^{+\infty} \frac{\rho^{2\beta+1}}{7\rho^{2\beta}+1}\,d\rho$$

STUDIAMOLO MEDIANTE CONFR. ASINTOTICO

$$f(x) = \frac{x^{2\beta+2}}{7x^{2\beta}+1} \qquad g(x) = x$$

$$\lim_{x\to+\infty} \frac{f(x)}{g(x)} = \lim_{x\to+\infty} \frac{x^{2\beta}}{7x^{2\beta}+1} = \lim_{x\to+\infty} \frac{1}{7+\frac{1}{x^{2\beta}}} = \frac{1}{7}$$

QUINDI PER $\alpha<0$

$$\int_1^{+\infty} \frac{\rho}{7+\rho^{2\alpha}}\, d\rho$$ SI COMPORTA COME $$\int_1^{+\infty} \rho\, d\rho$$ E DUNQUE DIVERGE

IN CONCLUSIONE

$$\int_1^{+\infty} \frac{\rho}{7+\rho^{2\alpha}}\, d\rho \begin{cases} \text{CONVERGE PER } \alpha>1 \\ \text{DIVERGE PER } \alpha\le 1 \end{cases}$$

E COSÍ ANCHE

$$\int_{\mathbb{R}^2} \frac{1}{7+(x^2+y^2)^{\alpha}}\, dx\, dy$$