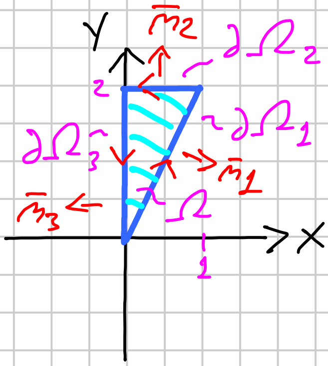
BY GIMUSI 03/07/15

$$\begin{cases} \partial\Omega : (\delta, 2\delta),\ \delta \in [0,1] \cup y=2 \cup x=0 \\ f(x,y) = x^2 \end{cases}$$



MODO 1

$$\int_\Omega x^2\,dx\,dy = \int_0^1\int_{2x}^2 x^2\,dy\,dx = \int_0^1 x^2(2-2x)\,dx =$$

$$= \int_0^1 (2x^2-2x^3)\,dx = \left[\frac{2}{3}x^3 - \frac{1}{2}x^4\right]_0^1 = \frac{2}{3} - \frac{1}{2} = \frac{1}{6}$$

MODO 2

$$\int_\Omega x^2\,dx\,dy = \int_0^2\int_0^{y/2} x^2\,dx\,dy = \frac{1}{3}\int_0^2 \left[x^3\right]_0^{y/2} dy =$$

$$= \frac{1}{24}\int_0^2 y^3\,dy = \frac{1}{24}\left[\frac{1}{4}y^4\right]_0^2 = \frac{1}{24}\cdot\frac{1}{4}\cdot 16 = \frac{1}{6}$$

MODO 3

$$\vec{E} = (0, yx^2) \qquad \operatorname{div}\vec{E} = x^2 \quad \rightsquigarrow \int_\Omega x^2\,dx\,dy = \int_{\partial\Omega} \vec{E}\cdot\vec{n}\,ds$$

$$\int_{\partial\Omega}\vec{E}\cdot\vec{n}\,ds = \int_{\partial\Omega_1}\vec{E}\cdot\vec{n}_1\,ds + \int_{\partial\Omega_2}\vec{E}\cdot\vec{n}_2\,ds + \underbrace{\cancel{\int_{\partial\Omega_3}\vec{E}\cdot\vec{n}_3\,ds}}_{=0} =$$

$$\int_{\partial\Omega_1}\vec{E}\cdot\vec{n}_1\,ds = \int_{\partial\Omega_1} -yx^2\,dx = \int_0^1 -2\delta^3\,d\delta = \left[-\frac{\delta^4}{2}\right]_0^1 = -\frac{1}{2}$$

$$\int_{\partial\Omega_2}\vec{E}\cdot\vec{n}_2\,ds = \int_{\partial\Omega_2} -yx^2\,dx = -\int_0^1 -2\delta^2\,d\delta = \left[\frac{2}{3}\delta^3\right]_0^1 = \frac{2}{3}$$

$$\rightsquigarrow \int_\Omega x^2\,dx\,dy = \int_{\partial\Omega}\vec{E}\cdot\vec{n}\,ds = -\frac{1}{2} + \frac{2}{3} = \frac{-3+4}{6} = \frac{1}{6}$$