$f(x,y,z) = x^2 - xy^2 + z^2 \qquad D: \; x^2+y^2+z^2-1=0$

D COMPATTO $\rightsquigarrow$ f AMMETTE MAX E MIN (T. DI WEIERSTRASS)

MODO 1 - SOSTITUZIONE

$x^2+y^2+z^2-1=0 \rightsquigarrow z^2 = 1-y^2-x^2$

$f(x,y,z) = g(x,y) = 1-y^2-xy^2 \qquad \Omega = \{(x,y): x^2+y^2 \le 1\}$

PUNTI STAZIONARI

$$\begin{cases} f_x = -y^2 = 0 \\ f_y = -2y-2yx = 0 \end{cases} \quad \begin{cases} y = 0 \\ y(x+1) = 0 \end{cases} \rightsquigarrow P_0' = (x,0) \quad -1<x<1$$

STUDIO SUL BORDO $\partial\Omega: \; \Phi = x^2+y^2-1=0$

SOSTITUZIONE: $y^2 = 1-x^2$

$g(x,y) = g(x) = 1-(1-x^2)-x(1-x^2) = \cancel{1}-\cancel{1}+x^2-x+x^3$

$g'(x) = 3x^2+2x-1=0 \qquad x = \dfrac{-2\pm\sqrt{5+12}}{6} = \begin{cases} -1 \in \text{BORDO} \\ 1/3 \end{cases}$

$$\begin{cases} x = 1/3 \rightsquigarrow y^2 = 8/9 \qquad y = \pm 2\sqrt{2}/3 \qquad P_{1,2}' = (1/3, \pm 2\sqrt{2}/3) \\ \text{BORDO: } x = \pm 1 \rightsquigarrow y = 0 \qquad P_{3,4}' = (\pm 1, 0) \end{cases}$$

$P_0' = (x,0) \; -1<x<1 \rightsquigarrow z^2 = 1-x^2 \rightsquigarrow P_0 = (x,0,\pm\sqrt{1-x^2}) \quad f(P_0) = 1$

$P_{3,4}' = (\pm 1, 0) \rightsquigarrow z^2 = 0 \rightsquigarrow P_{3,4} = (\pm 1, 0, 0) \quad f(P_3) = f(P_4) = 1$

$P_{1,2}' = (1/3, \pm 2\sqrt{2}/3) \rightsquigarrow z^2 = 0 \rightsquigarrow P_{1,2} = (1/3, \pm 2\sqrt{2}/3, 0)$

$f(P_1) = f(P_2) = \dfrac{1}{9} - \dfrac{1}{3}\dfrac{8}{9} = -5/27$

$$\rightsquigarrow \begin{cases} \text{MAX } f = 1 \quad P = (x,0,\pm\sqrt{1-x^2}) \quad -1 \le x \le 1 \\ \text{MIN } f = -5/27 \quad P_{1,2} = (1/3, \pm 2\sqrt{2}/3, 0) \end{cases}$$

MODO 2 – MOLT. DI LAGRANGE $\quad \Phi = x^2+y^2+z^2-1=0$

SISTEMA 1
$$\begin{cases} \Phi_x = 2x = 0 \\ \Phi_y = 2y = 0 \\ \Phi_z = 2z = 0 \\ x^2+y^2+z^2-1=0 \end{cases} \rightsquigarrow \begin{cases} x=0 \\ y=0 \\ z=0 \\ -1=0 \end{cases} \rightsquigarrow \emptyset$$

SISTEMA 2
$$\begin{cases} f_x = 2x-y^2 = \lambda\cdot 2x \\ f_y = -2xy = \lambda\cdot 2y \\ f_z = 2z = \lambda\cdot 2z \\ x^2+y^2+z^2-1=0 \end{cases} \rightsquigarrow 2z(\lambda-1)=0 \begin{cases} \lambda=1 \\ z=0 \end{cases}$$

$\lambda=1 \rightsquigarrow$
$$\begin{cases} 2x-y^2=2x \\ -2xy=2y \\ x^2+y^2+z^2-1=0 \end{cases} \begin{cases} y=0 \\ 0=0 \\ x^2+z^2=1 \end{cases} \rightsquigarrow P_0 = (x, 0, \pm\sqrt{1-x^2})$$

$z=0 \rightsquigarrow$
$$\begin{cases} 2x-y^2=2x\cdot\lambda \\ -2xy=2y\cdot\lambda \\ x^2+y^2-1=0 \end{cases} \rightsquigarrow 2y(\lambda+x)=0 \begin{cases} y=0 \\ \lambda=-x \end{cases}$$

$y=0$
$$\begin{cases} 2x=2x\cdot\lambda \\ x^2-1=0 \end{cases} \begin{cases} \lambda=\pm 1 \\ x=\pm 1 \end{cases} \rightsquigarrow P_{1,2}=(\pm 1, 0, 0)$$

$\lambda=-x$
$$\begin{cases} 2x-y^2=-2x^2 \\ x^2+y^2-1=0 \end{cases} \begin{cases} 2x^2+2x+x^2-1=0 \\ x^2+y^2-1=0 \end{cases} \begin{cases} 3x^2+2x-1=0 \\ x^2+y^2-1=0 \end{cases}$$
$$x = \frac{-2\pm\sqrt{5+12}}{6} \begin{cases} -1 \rightsquigarrow P_2 \\ 1/3 \rightsquigarrow y = \pm 2\sqrt{2}/3 \end{cases}$$

$$\rightsquigarrow P_{3,5} = (1/3, \pm 2\sqrt{2}/3, 0)$$

$$\rightsquigarrow \begin{cases} \text{MAX } f=1 \quad P=(x,0,\pm\sqrt{1-x^2}) \quad -1\le x\le 1 \\ \text{MIN } f=-5/27 \quad P_{1/2}=(1/3, \pm 2\sqrt{2}/3, 0) \end{cases}$$