

3 Luglio 2010

Titolo nota

01/07/2014

Es 3) "Convergenza Integrale"

$$B = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 \leq 1\} \rightarrow \{0 \leq \rho \leq 1, 0 \leq \theta \leq 2\pi\}$$

$$\int_B \frac{\sin^2(x+y)}{x^2+y^2} dx dy = \int_0^1 d\rho \int_0^{2\pi} \frac{\sin^2(\rho(\cos\theta + \sin\theta))}{\rho^2(\underbrace{\cos^2\theta + \sin^2\theta}_{=1})} \cdot \underbrace{\rho}_{\cancel{\rho}} d\theta =$$

$$= \int_0^1 \int_0^{2\pi} \frac{\sin^2[\rho(\cos\theta + \sin\theta)]}{\rho} d\rho d\theta = \star'$$

$$-\rho \leq \rho(\cos\theta + \sin\theta) \leq \rho \rightarrow \sin^2[\rho(\overbrace{\cos\theta + \sin\theta}^{\leq 1})] = C > 0$$

$$\star' = 2\pi \cdot C \int_0^1 \frac{1}{\rho} d\rho.$$

Nel nostro caso:

$$\int_0^L \frac{1}{\rho^\alpha} d\rho \rightarrow$$

$$\frac{1}{\rho} = \frac{1}{\rho^1} \Rightarrow 1 < 2 \rightarrow \text{CONVERGENTE.}$$

CONVERGENTE SE  $\alpha < 2$