$$f(x,y) = x\, e^{-(x^2+y^2)}$$

STUDIO CON WEIERSTRASS

1) $f(x,y)$ È CONTINUA SU $\mathbb{R}^2$

2) $f(0,0) = 0$

3) $\lim\limits_{x^2+y^2 \to +\infty} f(x,y) = \lim\limits_{\rho \to +\infty} \rho \cos\theta\, e^{-\rho^2} = 0$

INFATTI: $\underbrace{-\rho e^{-\rho^2}}_{\to 0} \le \underbrace{\rho \cos\theta\, e^{-\rho^2}}_{\to 0} \le \underbrace{\rho e^{-\rho^2}}_{\to 0}$

$\leadsto f(x,y)$ AMMETTE MAX(=SUP) E MIN(=INF)
(≡ É LIMITATA IN $\mathbb{R}^2$)

INOLTRE:

5) $f(x,y) \begin{cases} >0 & \text{PER } x>0 \\ <0 & \text{PER } x<0 \end{cases}$ $\quad$ <u>OSS</u> $e^{-(x^2+y^2)} > 0$ IN $\mathbb{R}^2$

$\leadsto$ MAX $f > 0$ $\quad$ MIN $f < 0$

PUNTI STAZIONARI

$$\begin{cases} f_x = \overset{\neq 0}{\cancel{e^{-(x^2+y^2)}}} + x(-2x)\, \overset{\neq 0}{\cancel{e^{-(x^2+y^2)}}} = 0 \\ f_y = -2y\, \overset{\neq 0}{\cancel{e^{-(x^2+y^2)}}} = 0 \end{cases} \qquad \begin{cases} 1-2x^2=0 \\ y=0 \end{cases}$$

$$\begin{cases} x = \pm\sqrt{2}/2 \\ y=0 \end{cases} \leadsto P_1 = \left(\frac{\sqrt{2}}{2}, 0\right) \quad P_2 = \left(-\frac{\sqrt{2}}{2}, 0\right)$$

$$\begin{cases} f(P_1) = \frac{\sqrt{2}}{2} e^{-\left(\frac{1}{2}\right)} = \frac{\sqrt{2}}{2\sqrt{e}} > 0 \\ f(P_2) = -\frac{\sqrt{2}}{2} e^{-\left(\frac{1}{2}\right)} = \frac{-\sqrt{2}}{2\sqrt{e}} < 0 \end{cases}$$

$$\leadsto \begin{cases} \text{SUP} f = \text{MAX} f = \frac{\sqrt{2}}{2\sqrt{e}} \quad \text{IN} \quad P_1 = (\sqrt{2}/2, 0) \\ \text{INF} f = \text{MIN} f = \frac{-\sqrt{2}}{2\sqrt{e}} \quad \text{IN} \quad P_2 = (-\sqrt{2}/2, 0) \end{cases}$$