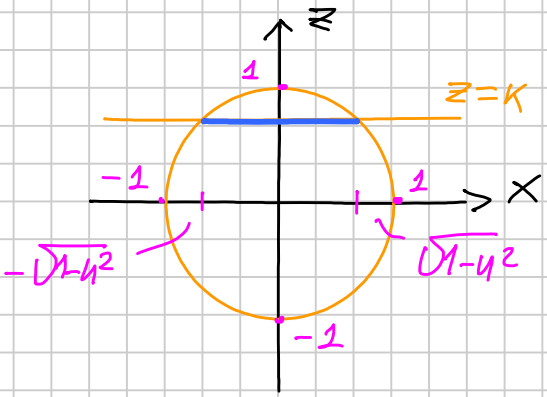
$$\begin{cases} \Omega = \{x^2+y^2 \le 1,\ x^2+z^2 \le 1\} \\ \vec{E} = (x+3y,\ z^3 y,\ x+z) \end{cases}$$

(ASSE Z, ASSE Y) $\equiv$ INTERSEZIONE CILINDRI



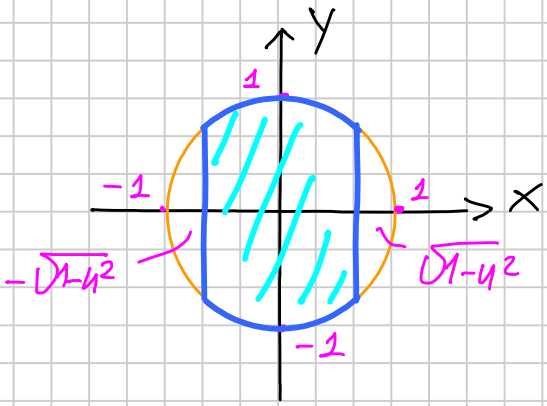

$$\operatorname{div}\vec{E} = 1 + z^3 + 1 = 2 + z^3$$

$$\int_{\partial\Omega} \vec{E}\cdot\vec{n}\,d\sigma = \int_{\Omega} \operatorname{div}\vec{E}\,dx\,dy\,dz =$$

$$= \int_{\Omega} (2+z^3)\,dx\,dy\,dz = \int_{\Omega} 2\,dx\,dy\,dz +$$

$$+ \int_{\Omega} z^3\,dx\,dy\,dz = 2\int_{\Omega} dx\,dy\,dz =$$

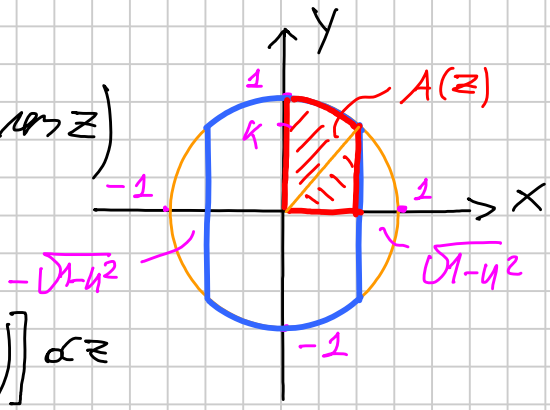
($\int_{\Omega} z^3\,dx\,dy\,dz = 0$ per SIMMETRIA)




## MODO 1 - PIANI $z=k$

$$= 2\int_{-1}^{1}\int_{-\sqrt{1-z^2}}^{\sqrt{1-z^2}}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} dy\,dx\,dz = 16\int_{0}^{1}\int_{0}^{\sqrt{1-z^2}}\int_{0}^{\sqrt{1-x^2}} dy\,dx\,dz =$$

$$A(z) = \int_0^{\sqrt{1-z^2}}\int_0^{\sqrt{1-x^2}} dy\,dx = \frac{z}{2}\sqrt{1-z^2} + \left(\frac{\pi}{4} - \frac{1}{2}\arcsin z\right)$$




$$= 16\int_0^1 A(z)\,dz = 16\int_0^1 \left[\frac{z}{2}\sqrt{1-z^2} + \left(\frac{\pi}{4} - \frac{1}{2}\arcsin z\right)\right] dz$$

$$= 16\cdot\frac{1}{2}\left[-\frac{1}{3}(1-z^2)^{3/2}\right]_0^1 + 16\cdot\frac{\pi}{4} - \frac{16}{2}\left[\sqrt{1-z^2} + z\arcsin z\right]_0^1 =$$

$$= 8\left(\frac{1}{3}\right) + 4\pi - 8\left(\frac{\pi}{2} - 1\right) = \frac{8}{3} + 4\pi - 4\pi + 8 = \frac{32}{3}$$

## METODO 2 - PIANI $x = k$

$$= 2\int_{-2}^{2} A(x)\,dx = 2\int_{-1}^{1} 5(1-x^2)\,dx =$$

$$= 16\int_{0}^{1}(1-x^2)\,dx = 16\left[x - \frac{x^3}{3}\right]_0^1 =$$

$$= 16\left(1-\frac{1}{3}\right) = \frac{32}{3}$$