BY GIMUSI 14/06/14

DA TEST 42

2.8) $q(x,y,z) = z^2 + 3yz + 5xz$

$$A = \begin{pmatrix} 0 & 0 & 5/2 \\ 0 & 0 & 3/2 \\ 5/2 & 3/2 & 1 \end{pmatrix}$$

1) COMPLETAMENTO DEI QUADRATI

$$\left(z+\frac{3}{2}y+\frac{5}{2}x\right)^2 - \frac{9}{4}y - \frac{25}{4}x^2 - \frac{15}{2}xy = \left(z+\frac{3}{2}y+\frac{5}{2}x\right)^2 - \left(\frac{5}{2}x+\frac{3}{2}y\right)^2$$

$$\begin{cases} q(v) \geq 0 & v = \left(\frac{2}{5}\delta, -\frac{2}{3}\delta, \delta\right) \\ q(v) \leq 0 & v = (\delta, \delta, -4\delta) \\ q(v) = 0 & v = \left(\frac{2}{5}\delta, -\frac{2}{3}\delta, 0\right) \end{cases} \rightsquigarrow \quad m^+ = 1,\ m^- = 1,\ m^0 = 1$$

SYLVESTERIZZAZIONE

$$v_1 = \left(\frac{2}{5}, \frac{-2}{3}, 1\right) \rightsquigarrow v_1^T A v_1 = v_1^T \begin{pmatrix} 5/2 \\ 3/2 \\ 1 \end{pmatrix} = 1 \rightsquigarrow w_1 = v_1$$

$$v_2 = (1, 1, -4) \qquad v_2^T A v_1 = \frac{5}{2} + \frac{3}{2} - 4 = 0$$

$$v_2^T A v_2 = v_2^T \begin{pmatrix} -10 \\ -6 \\ 0 \end{pmatrix} = -16 \rightsquigarrow w_2 = \frac{v_2}{\sqrt{16}} = \left(\frac{1}{4}, \frac{1}{4}, -1\right)$$

$$v_3 = (2/5, -2/3, 0) \qquad v_3^T A v_1 = 0 \qquad v_3^T A v_2 = 0$$

$$v_3^T A v_3 = v_3^T \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} = 0 \rightsquigarrow w_3 = (6, -10, 0)$$

$$M = \begin{array}{c} \begin{array}{ccc} w_1 & w_2 & w_3 \end{array} \\ \begin{pmatrix} 2/5 & 1/4 & 6 \\ -2/3 & 1/4 & -10 \\ 1 & -1 & 0 \end{pmatrix} \end{array}$$

$$S = M^\delta A M = M^\delta \overset{A}{\begin{pmatrix} 0 & 0 & 5/2 \\ 0 & 0 & 3/2 \\ 5/2 & 3/2 & 1 \end{pmatrix}} \overset{M}{\begin{pmatrix} 2/5 & 1/4 & 6 \\ -2/3 & 1/4 & -10 \\ 1 & -1 & 0 \end{pmatrix}} =$$

$$= \overset{M^\delta}{\begin{pmatrix} 2/5 & -2/3 & 1 \\ 1/4 & 1/4 & -1 \\ 6 & -10 & 0 \end{pmatrix}} \overset{AM}{\begin{pmatrix} 5/2 & -5/2 & 0 \\ 3/2 & -3/2 & 0 \\ 1 & 0 & 0 \end{pmatrix}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

2) AUTOVALORI E AUTOVETTORI

$$|A-\lambda I| = \begin{vmatrix} -\lambda & 0 & 5/2 \\ 0 & -\lambda & 3/2 \\ 5/2 & 3/2 & 1-\lambda \end{vmatrix} = \lambda^2(1-\lambda) + \left(\frac{5}{2}\right)^2 \lambda + \left(\frac{3}{2}\right)^2 \lambda = 0$$

$$\lambda\left(\lambda - \lambda^2 + \frac{35}{4}\right) = 0 \quad \lambda_3 = 0 \quad \lambda_{1,2} = \frac{1 \pm \sqrt{35}}{2}$$

$$\lambda_1 : \begin{pmatrix} \frac{-1-\sqrt{35}}{2} & 0 & \frac{5}{2} \\ 0 & \frac{-1-\sqrt{35}}{2} & \frac{3}{2} \\ \frac{5}{2} & \frac{3}{2} & \frac{1-\sqrt{35}}{2} \end{pmatrix} v = 0 \quad \begin{pmatrix} \frac{-1-\sqrt{35}}{2} & 0 & \frac{5}{2} \\ 0 & \frac{-1-\sqrt{35}}{2} & \frac{3}{2} \\ \frac{-1-\sqrt{35}}{2} & \frac{3(-1-\sqrt{35})}{10} & \frac{17}{5} \end{pmatrix} v = 0$$

$$\begin{pmatrix} \frac{-1-\sqrt{35}}{2} & 0 & \frac{5}{2} \\ 0 & \frac{-1-\sqrt{35}}{2} & \frac{3}{2} \\ 0 & \frac{3(-1-\sqrt{35})}{10} & \frac{9}{10} \end{pmatrix} v = 0 \quad \begin{pmatrix} -1-\sqrt{35} & 0 & 5 \\ 0 & -1-\sqrt{35} & 3 \\ 0 & -1-\sqrt{35} & 3 \end{pmatrix} v = 0$$

$$\begin{cases} v_1 = (5,\ 3,\ 1+\sqrt{35}) \quad \|v_1\| = (25+9+36+2\sqrt{35})^{1/2} = \sqrt{70+2\sqrt{35}} \\ \sqrt{\lambda_1} = \sqrt{\frac{1+\sqrt{35}}{2}} \end{cases} \rightsquigarrow w_1 = \frac{1}{\sqrt{\lambda_1}} \frac{v_1}{\|v_1\|} \qquad w_1^\delta w_1 = 1/\lambda_1$$

$$w_1^\delta A w_1 = \lambda_1 w_1^\delta w_1 = \lambda_1/\lambda_1 = 1$$

$\rightsquigarrow$ ANALOGAMENTE PER W_2 E W_3