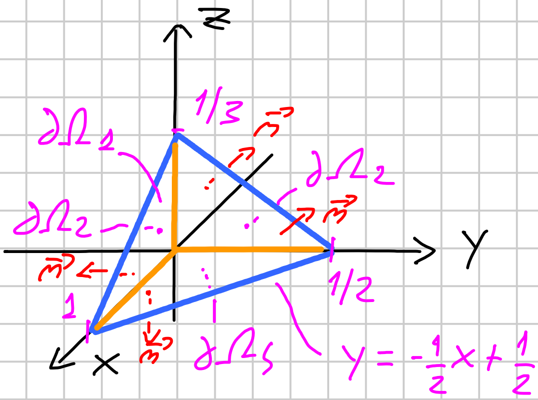
BY GIMUSI 14/06/14

$$\begin{cases} \partial\Omega : \underbrace{\{x+2y+3z=1,\ x\geq 0,\ y\geq 0,\ z\geq 0\}}_{\partial\Omega_1} \cup \underbrace{\{x=0\}}_{\partial\Omega_2} \cup \underbrace{\{y=0\}}_{\partial\Omega_3} \cup \underbrace{\{z=0\}}_{\partial\Omega_4} \\ \Phi = y \end{cases}$$



MODO 1 - CALCOLO DIRETTO

$$\int_\Omega y \, dx\,dy\,dz = \int_0^1 \int_0^{-\frac{1}{2}x+\frac{1}{2}} \int_0^{\frac{1}{3}-\frac{x}{3}-\frac{2}{3}y} y \, dz\,dy\,dx =$$

$$= \int_0^1 \int_0^{-\frac{1}{2}x+\frac{1}{2}} y\left(\frac{1}{3}-\frac{x}{3}-\frac{2}{3}y\right) dy\,dx = \frac{1}{3}\int_0^1 \left[y^2/2 - \frac{xy^2}{2} - \frac{2}{3}y^3 \right]_0^{-\frac{1}{2}x+\frac{1}{2}} dx =$$

$$= \frac{1}{6}\int_0^1 \left[\left(-\frac{1}{2}x+\frac{1}{2}\right)^2 - x\left(-\frac{1}{2}x+\frac{1}{2}\right)^2 - \frac{4}{3}\left(-\frac{1}{2}x+\frac{1}{2}\right)^3 \right] dx =$$

$$= \frac{1}{24}\int_0^1 \left[x^2+1-2x-x^3-x+2x^2 - \cancel{\frac{16}{24}}\,\frac{2}{3}\left(-x^3-3x+3x^2+1\right)\right] dx =$$

$$= \frac{1}{24}\int_0^1 \left[(-x^3+3x^2-3x+1) - \frac{2}{3}(-x^3+3x^2-3x+1)\right] dx =$$

$$= \frac{1}{72}\int_0^1 (-x^3+3x^2-3x+1)\,dx = \frac{1}{72}\left[-\frac{x^4}{4}+x^3-\frac{3}{2}x^2+x\right]_0^1 = \frac{1}{72}\left(-\frac{1}{4}+1-\frac{3}{2}+1\right) =$$

$$= \frac{1}{72}\left(2-\frac{7}{4}\right) = \frac{1}{72}\cdot\frac{1}{4} = \frac{1}{288}$$

MODO 2 - FORMULA DI GAUSS-GREEN

$$\int_\Omega \operatorname{div} \vec{E} \, dx\,dy\,dz = \int_{\partial\Omega} \vec{E}\cdot\vec{n}\, d\sigma \quad , \quad \text{SIA } \vec{E} = (0, 0, zy) \rightsquigarrow \operatorname{div}\vec{E} = y$$

$$\int_\Omega y\, dx\,dy\,dz = \int_{\partial\Omega_1} \vec{E}\cdot\vec{n}\, d\sigma + \cancel{\int_{\partial\Omega_2} 0 \cdot d\sigma} + \cancel{\int_{\partial\Omega_3} 0 \cdot d\sigma} + \cancel{\int_{\partial\Omega_4} 0 \cdot d\sigma} =$$

$$\partial\Omega_2 : \left(x, y, \frac{1}{3}-\frac{x}{3}-\frac{2}{3}y\right) \quad x\in[0,1] \quad 0 \leq y \leq -\frac{1}{2}x+\frac{1}{2}$$

$$\begin{pmatrix} 1 & 0 & -1/3 \\ 0 & 1 & -2/3 \end{pmatrix} \rightsquigarrow n_1 = \frac{1}{3} \quad n_2 = \frac{2}{3} \quad n_3 = 1$$

VETTORE NORMALE : $\vec{N} = \left(\frac{1}{3}, \frac{2}{3}, 1\right)$

COME PRIMA

$$\int_{\partial\Omega_2} \vec{E} \cdot \vec{n} \, d\sigma = \int_{\partial\Omega_2} \vec{E} \cdot \vec{N} \, dx \, dy = \int_0^1 \int_0^{-\frac{1}{2}x + \frac{1}{2}} y\left(\frac{1}{3} - \frac{x}{3} - \frac{2}{3}y\right) dy \, dx = \dots = \frac{1}{288}$$