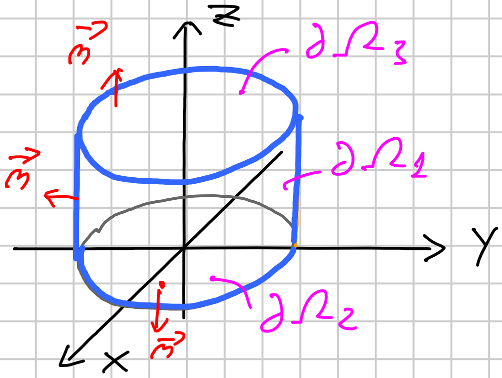
BY GIMUSI 15/06/15

$$\begin{cases} \partial\Omega : \{ \overset{\partial\Omega_1}{(\cos\theta, \operatorname{sen}\theta, v), (\theta, v) \in [0, 2\pi]\times[0,1]} \} \cup \overset{\partial\Omega_2}{\{z=0\}} \cup \overset{\partial\Omega_3}{\{z=1\}} \\ \Phi = x^2 \end{cases}$$



MODO 1 - CALCOLO DIRETTO

$$\int_\Omega x^2 \, dx\,dy\,dz = \int_0^1 \int_0^1 \int_0^{2\pi} \rho^2 \cos^2\theta \, \rho \, d\theta\, d\rho\, dz =$$

$$= \int_0^1 \int_0^1 \pi \rho^3 \, d\rho\, dz = \pi \int_0^1 [\rho^5/5]_0^1 \, dz = \frac{\pi}{5}$$

MODO 2 - FORMULA DI GAUSS - GREEN

$$\int_\Omega \operatorname{div}\vec{E} \, dx\,dy\,dz = \int_{\partial\Omega} \vec{E}\cdot\vec{n}\, d\sigma \quad , \quad \text{SIA } \vec{E} = (0, 0, zx^2) \rightsquigarrow \operatorname{div}\vec{E} = x^2$$

$$\int_\Omega x^2 \, dx\,dy\,dz = \cancel{\int_{\partial\Omega_1} 0\cdot d\sigma} + \cancel{\int_{\partial\Omega_2} 0\cdot d\sigma} + \int_{\partial\Omega_3} x^2 \, d\sigma = \int_0^{2\pi}\int_0^1 \rho^2 \cos^2\theta \, \rho \, d\rho\, d\theta =$$

$$= \int_0^{2\pi} \cos^2\theta \, [\rho^5/5]_0^1 \, d\theta = \frac{1}{5}\int_0^{2\pi} \cos^2\theta \, d\theta = \frac{\pi}{5}$$