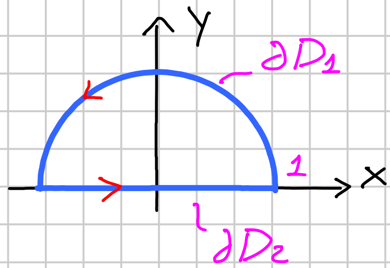
BY GIMUSI 07/06/15

$$\partial D: \begin{cases} y=0 \\ \gamma(\vartheta)=(\cos\vartheta, \sin\vartheta), \ \vartheta\in[0,\pi] \end{cases}$$



$$\Phi = y$$

$$\int_D \operatorname{div} \vec{E} \, dx\,dy = \int_{\partial D} \vec{E}\cdot\vec{n} \, ds = \int_{\partial D} A\,dy - B\,dx$$

$$\vec{E} = (\overset{A}{xy}, \overset{B}{0}) \qquad \operatorname{div} \vec{E} = A_x + B_y = y = \Phi$$

$$\rightsquigarrow \int_D y\,dx\,dy = \int_{\partial D} A\,dy = \int_{\partial D_1} A\,dy + \overbrace{\cancel{\int_{\partial D_2} A\,dy}}^{=0}$$

$$\int_{\partial D_1} A\,dy = \int_{\partial D_1} xy\,dy = \int_0^{\pi} \sin\vartheta\cos\vartheta \cdot \cos\vartheta \, d\vartheta = -\int_0^{\pi} \cos^2\vartheta \, d(\cos\vartheta) =$$

$$= -\frac{1}{3}\left[\cos^3\vartheta\right]_0^{\pi} = -\frac{1}{3}(-1-1) = \frac{2}{3}$$

<u>OSS</u> $\int_D y\,dx\,dy = y_G \cdot AREA = \frac{4}{3\pi}\cdot\frac{\pi}{2} = \frac{2}{3}$