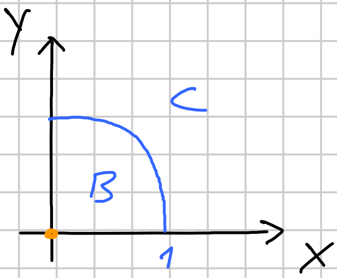
BY GIMUSI 03/06/15

$$\int_A \frac{\operatorname{arctg} x}{(x^2+y^2)^\alpha}\,dx\,dy \qquad A = B \cup C$$



$$= \int_B \frac{\operatorname{arctg} x}{(x^2+y^2)^\alpha}\,dx\,dy + \int_C \frac{\operatorname{arctg} x}{(x^2+y^2)^\alpha}\,dx\,dy$$

$$\int_B \frac{\operatorname{arctg} x}{(x^2+y^2)^\alpha}\,dx\,dy \qquad x \to 0^+ \quad \operatorname{arctg} x \sim x \quad \rightsquigarrow \text{ STESSO COMPORTAMENTO}$$

DI

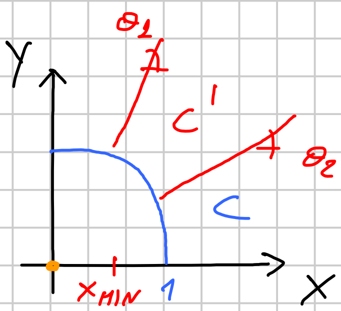
$$\int_B \frac{x}{(x^2+y^2)^\alpha}\,dx\,dy = \int_0^{\pi/2}\int_0^1 \frac{\rho\cos\theta}{\rho^{2\alpha}}\,\rho\,d\rho\,d\theta = \int_0^{\pi/2}\cos\theta\,d\theta \int_0^1 \frac{d\rho}{\rho^{2\alpha-2}} =$$

$$= \int_0^1 \frac{d\rho}{\rho^{2\alpha-2}} < +\infty \text{ PER } 2\alpha-2<1 \rightsquigarrow \text{ CONVERGE } \iff \alpha < 3/2$$

$$\int_C \frac{\operatorname{arctg} x}{(x^2+y^2)^\alpha}\,dx\,dy \le \frac{\pi}{2}\int_C \frac{1}{(x^2+y^2)^\alpha}\,dx\,dy = \frac{\pi}{2}\int_0^{\pi/2}\int_0^{+\infty} \frac{1}{\rho^{2\alpha-1}}\,d\rho\,d\theta =$$

$$= \frac{\pi^2}{4}\int_0^{+\infty} \frac{1}{\rho^{2\alpha-1}}\,d\rho < +\infty \text{ PER } 2\alpha-1>1 \rightsquigarrow \text{ CONVERGE PER } \alpha>1$$

PER $\alpha \le 1$



$$\int_C \frac{\operatorname{arctg} x}{(x^2+y^2)^\alpha}\,dx\,dy \ge \int_{C'} \frac{\operatorname{arctg} x_{MIN}}{(x^2+y^2)^\alpha}\,dx\,dy =$$

$$= \operatorname{arctg} x_{MIN}\cdot(\theta_1-\theta_2)\int_0^{+\infty} \frac{1}{\rho^{2\alpha-1}}\,d\rho = +\infty \text{ PER } 2\alpha-1 \le 1 \rightsquigarrow \alpha \le 1$$

$$\rightsquigarrow \int_A \frac{\operatorname{arctg} x}{(x^2+y^2)^\alpha}\,dx\,dy \quad \text{CONVERGE} \iff 1 < \alpha < \frac{3}{2}$$