$\mathbb{R}_{\leq 2}[x]$	$\mathbb{R}^3$	$x^2+3 \to (0,1,2)$ $x \to (-2,-1,5)$ $x^2-3 \to (1,0,1)$

BY GIMUSI 27/05/16

ESISTENZA E UNICITÀ

BASE CANONICA DI $\mathbb{R}_{\leq 2}[x]$: $\{1, x, x^2\}$

$$\begin{cases} v_1 = x^2+3 = (3,0,1) \\ v_2 = x \quad = (0,1,0) \\ v_3 = x^2-3 = (-3,0,1) \end{cases} \qquad \begin{pmatrix} 3 & 0 & 1 \\ 0 & 1 & 0 \\ -3 & 0 & 1 \end{pmatrix} \rightsquigarrow \begin{pmatrix} 3 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} \rightsquigarrow v_1, v_2, v_3 \text{ SONO UNA BASE}$$

$\rightsquigarrow$ L'APPLICAZIONE LINEARE ESISTE ED È UNICA

DETERMINAZIONE MATRICE ASSOCIATA IN BASE CANONICA

MODO 1 "A OCCHIO"

SAPPIAMO CHE:

$$\begin{cases} v_1 = x^3+3 \to (0,1,2) \\ v_2 = x \to (-2,-1,5) \\ v_3 = x^2-3 \to (1,0,1) \end{cases} \rightsquigarrow \begin{cases} v_1+v_2 = 2x^2 \to (1,1,3) \\ v_1 - v_2 = 6 \to (-1,1,1) \\ v_2 = x \to (-2,-1,5) \end{cases}$$

$$\rightsquigarrow \begin{cases} (v_1+v_2)/2 = x^2 \to (1/2, 1/2, 3/2) \\ (v_1-v_2)/6 = 1 \to (-1/6, 1/6, 1/6) \\ v_2 = x \to (-2,-1,5) \end{cases} \equiv \begin{cases} (0,0,1) \to (1/2,1/2,3/2) \\ (1,0,0) \to (-1/6,1/6,1/6) \\ (0,1,0) \to (-2,-1,5) \end{cases}$$

PERTANTO

$$A = \begin{array}{c} \begin{array}{ccc} 1 & x & x^2 \end{array} \\ \begin{pmatrix} -1/6 & -2 & 1/2 \\ 1/6 & -1 & 1/2 \\ 1/6 & 5 & 3/2 \end{pmatrix} \end{array}$$

MODO 2 "CAMBIO DI BASE"

NELLA BASE "NON CANONICA" $\{v_1, v_2, v_3\}$ IN PARTENZA CON BASE CANONICA IN ARRIVO LA MATRICE ASSOCIATA È:

$$\hat{A} = \begin{array}{c} \begin{array}{ccc} x^2+3 & x & x^2-3 \end{array} \\ \begin{pmatrix} 0 & -2 & 1 \\ 1 & -1 & 0 \\ 2 & 5 & 1 \end{pmatrix} \end{array}$$

MATRICE DI CAMBIO BASE DI PARTENZA $\underset{\text{BASE } \mathcal{E}}{(v_1, v_2, v_3)} \rightsquigarrow \underset{\text{CANONICA}}{(1, x, x^2)}$

$$M = \begin{pmatrix} 3 & 0 & -3 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} \qquad \overset{\text{CANONICA}}{V_e} = M \underset{\text{BASE } \mathcal{E}}{V_\mathcal{E}} \rightsquigarrow V_\mathcal{E} = M^{-1} V_e$$

$$\rightsquigarrow W_e = \hat{A} V_\mathcal{E} = \hat{A} M^{-1} V_e = A V_e$$

$$\rightsquigarrow A = \hat{A} M^{-1} \qquad M^{-1} = \frac{1}{6} \begin{pmatrix} 1 & -0 & -1 \\ -0 & 6 & -0 \\ 3 & -0 & 3 \end{pmatrix}^T = \frac{1}{6} \begin{pmatrix} 1 & 0 & 3 \\ 0 & 6 & 0 \\ -1 & 0 & 3 \end{pmatrix}$$

$$A = \frac{1}{6} \begin{pmatrix} -1 & -12 & 3 \\ 1 & -6 & 3 \\ 1 & 30 & 9 \end{pmatrix} = \begin{pmatrix} -1/6 & -2 & 1/2 \\ 1/6 & -1 & 1/2 \\ 1/6 & 5 & 3/2 \end{pmatrix}$$

MODO 3 "BOVINO"

SI DEVE TROVARE A TALE CHE:

$$A \begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix} \qquad A \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} -2 \\ -1 \\ 5 \end{pmatrix} \qquad A \begin{pmatrix} -3 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$

= 9 EQUAZIONI IN 9 INCOGNITE

MODO 5 "RAPUANO'S METHOD" (EQ. A MODO 1)

$$\left(\begin{array}{ccc|ccc} 3 & 0 & 1 & 0 & 1 & 2 \\ 0 & 1 & 0 & -2 & -1 & 5 \\ -3 & 0 & 1 & 1 & 0 & 1 \end{array}\right) \rightsquigarrow \left(\begin{array}{ccc|ccc} 3 & 0 & 1 & 0 & 1 & 2 \\ 0 & 1 & 0 & -2 & -1 & 5 \\ 0 & 0 & 2 & 1 & 1 & 3 \end{array}\right)$$

$$\left(\begin{array}{ccc|ccc} 3 & 0 & 0 & -1/2 & 1/2 & 1/2 \\ 0 & 1 & 0 & -2 & -1 & 5 \\ 0 & 0 & 1 & 1/2 & 1/2 & 3/2 \end{array}\right) \rightsquigarrow \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -1/6 & 1/6 & 1/6 \\ 0 & 1 & 0 & -2 & -1 & 5 \\ 0 & 0 & 1 & 1/2 & 1/2 & 3/2 \end{array}\right)$$

$$\rightsquigarrow A = \begin{pmatrix} -1/6 & -2 & 1/2 \\ 1/6 & -1 & 1/2 \\ 1/6 & 5 & 3/2 \end{pmatrix}$$

DIMENSIONE E BASE DI KER E IMMAGINE

SI VERIFICA FACILMENTE CHE:

$$\begin{cases} \underset{\text{RANGO}(\hat{A})}{\overset{\shortparallel}{\text{RANGO}(A)}} = 3 \rightsquigarrow \text{DIM}(\text{KER}) = 0 \rightsquigarrow \text{DIM}(\text{IM}) = 3-0 = 3 \\ \text{BASE DI IM} = \text{COLONNE DI } \underset{(\hat{A})}{A} \quad (\text{OPPURE OGNI BASE IN } \mathbb{R}^3) \end{cases}$$