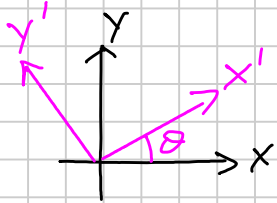


$$f(x, y) = x^2 + y^2$$

$$A: \begin{cases} x^2 + y^2 \leq 3 - xy \\ x^2 + y^2 + xy - 3 \leq 0 \end{cases}$$

BORDO DEL DOMINIO:  $\Phi = x^2 + y^2 + xy - 3 = 0$



$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x' \\ y' \end{pmatrix} \leadsto \begin{cases} x = x' \cos\theta + y' \sin\theta \\ y = -x' \sin\theta + y' \cos\theta \end{cases}$$

$$x^2 + y^2 + xy - 3 = (x' \cos\theta + y' \sin\theta)^2 + (-x' \sin\theta + y' \cos\theta)^2 + (x' \cos\theta + y' \sin\theta)(-x' \sin\theta + y' \cos\theta) - 3 =$$

SI OMETTONO GLI APICI

$$= x'^2 \cos^2\theta + y'^2 \sin^2\theta + 2x'y' \sin\theta \cos\theta + x'^2 \sin^2\theta + y'^2 \cos^2\theta - 2x'y' \sin\theta \cos\theta + -x'^2 \sin\theta \cos\theta + y'^2 \sin\theta \cos\theta + x'y' \cos^2\theta - x'y' \sin^2\theta - 3 =$$

$$= \left(1 - \frac{1}{2} \sin 2\theta\right) x'^2 + \left(1 + \frac{1}{2} \sin 2\theta\right) y'^2 + (\cos^2 2\theta) x'y' - 3 = 0$$

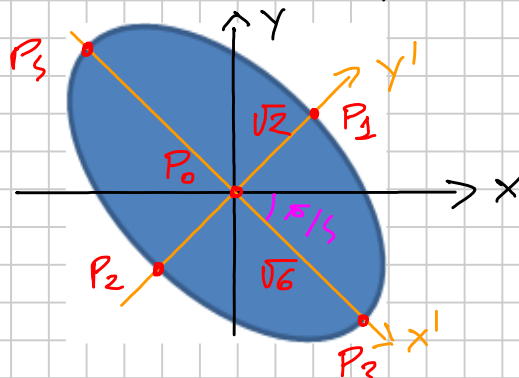
$$\cos 2\theta = 0 \quad 2\theta = \frac{\pi}{2} \quad \theta = \frac{\pi}{4}$$

EQ. DOPO ROT.  
ANTIORARIA DI  $\pi/4$



$$\leadsto \frac{x'^2}{2} + \frac{3}{2} y'^2 - 3 = 0 \quad \frac{x'^2}{6} + \frac{y'^2}{2} = 1$$

$\equiv$  ELLISSE CON ASSI RUOTATI DI  $\pi/4$



$\leadsto$  CON LE CURVE DI LIVELLO P.TI DI  
(MAX  $P_3 \equiv P_5$   
{ MIN  $P_0$

## STUDIO AL BORDO CON LA GRANGE:

$$\text{SISTEMA 1} \leadsto \begin{cases} 2x+y=0 \\ 2y+x=0 \\ x^2+y^2+xy-3=0 \end{cases} \quad \begin{cases} x=0 \\ y=0 \\ -3=0 \leadsto \text{IMPOSSIBILE} \end{cases}$$

$$\text{SISTEMA 2} \leadsto \begin{cases} 2x = \lambda(2x+y) \\ 2y = \lambda(2y+x) \\ x^2+y^2+xy-3=0 \end{cases} \leadsto \begin{cases} 2xy = \lambda(2x+y) \cdot y \\ 2yx = \lambda(2y+x) \cdot x \end{cases} \leadsto$$

$$\leadsto \cancel{2}(2x+y) \cdot y = \cancel{2}(2y+x) \cdot x \quad 2xy + y^2 = 2yx + x^2 \quad y^2 = x^2$$

( $\lambda = 0 \leadsto \text{sist. IMPOSS.}$ )

$$\leadsto y = \pm x \begin{cases} y=x \leadsto 3x^2-3=0 \quad x=\pm 1 \leadsto P_1, P_2 \\ y=-x \leadsto x^2-3=0 \quad x=\pm\sqrt{3} \leadsto P_3, P_5 \end{cases}$$

$$\begin{cases} P_1=(1,1) & P_2=(-1,1) & f(P_1)=f(P_2)=2 \\ P_3=(\sqrt{3},-\sqrt{3}) & P_5=(-\sqrt{3},\sqrt{3}) & f(P_3)=f(P_5)=6 \end{cases}$$

$$\leadsto \begin{cases} \text{MAX}=6 & \text{IN } P_3 \in P_5 \\ \text{MIN}=0 & \text{IN } P_0=(0,0) \end{cases}$$