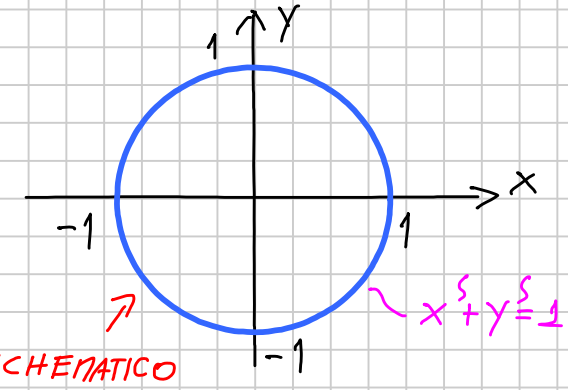
$f(x,y) = x^2 + 2y^2$ $\qquad A: \; x^5 + y^5 = 1$



$\rightsquigarrow$ MOLTIPLICATORI DI LAGRANGE

$$\Phi(x,y) = x^5 + y^5 - 1 = 0$$

$$\text{SISTEMA 1} \begin{cases} \Phi_x = 5x^3 = 0 \\ \Phi_y = 5y^3 = 0 \\ \Phi = 0 \end{cases} \rightsquigarrow \begin{cases} x = 0 \\ y = 0 \\ -1 = 0 \end{cases}$$

NESSUNA SOLUZIONE

$$\text{SISTEMA 2} \begin{cases} f_x = \lambda \Phi_x \\ f_y = \lambda \Phi_y \\ \Phi = 0 \end{cases} \rightsquigarrow \begin{cases} 2x = \lambda \cdot 5x^3 \\ 5y = \lambda \cdot 5y^3 \\ x^5 + y^5 - 1 = 0 \end{cases} \rightsquigarrow$$

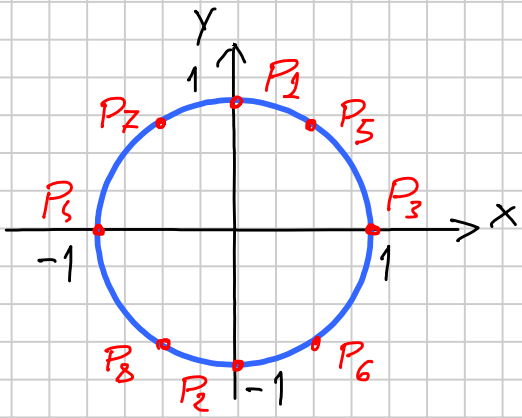
$$\rightsquigarrow \begin{cases} y^3 \cdot 2x = y^3 \cdot \lambda \cdot 5x^3 \\ x^3 \cdot (5y) = x^3 \cdot \lambda \cdot 5y^3 \end{cases} \rightsquigarrow xy^3 - 2x^3 y = 0$$

$$xy(y^2 - 2x^2) = 0 \rightsquigarrow \begin{cases} xy = 0 & A) \\ y^2 + 2x^2 = 0 & B) \end{cases}$$

$$A) \quad xy = 0 \rightsquigarrow \begin{cases} x = 0 \rightsquigarrow P_{1,2} = (0, \pm 1) \\ y = 0 \rightsquigarrow P_{3,4} = (\pm 1, 0) \\ x = y = 0 \rightsquigarrow \text{IMPOSSIBILE} \end{cases}$$

B) $y^2 - 2x^2 = 0 \rightsquigarrow x^5 + 5x^5 = 1 \quad x^5 = \frac{1}{5} \quad x = \pm \frac{1}{\sqrt[5]{5}}$

$\rightsquigarrow y^2 = \frac{2}{\sqrt{5}} \quad y = \pm \frac{\sqrt{2}}{\sqrt[5]{5}} \rightsquigarrow P_{5/6/7/8} = \left(\pm \frac{1}{\sqrt[5]{5}}, \pm \frac{\sqrt{2}}{\sqrt[5]{5}} \right)$



$f(P_1) = f(P_2) = 2 \cdot 1 = 2$

$f(P_3) = f(P_4) = 1$

$f(P_5) = f(P_6) = f(P_7) = f(P_8) = \frac{5}{\sqrt{5}} = \sqrt{5} > 2$

$\rightsquigarrow$ MAX $= \sqrt{5}$ IN P_5, P_6, P_7, P_8

MIN $= 1$ IN P_3 E P_4