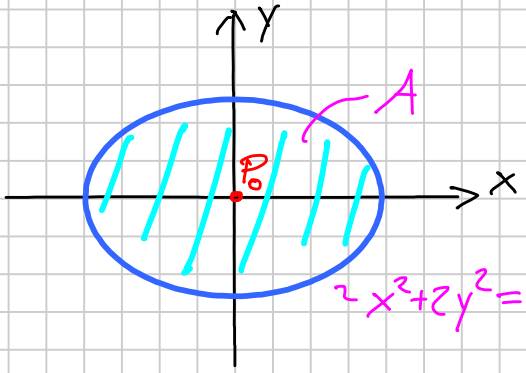
$f(x,y) = xy$ $\qquad A: x^2+2y^2 \le 5$



A É COMPATTO $\rightsquigarrow \exists$ MAX, MIN

1) PUNTI SING. INTERNI
$\rightsquigarrow$ NON CI SONO

2) PUNTI STAZIONARI INTERNI

$$\begin{cases} f_x = y = 0 \\ f_y = x = 0 \end{cases} \rightsquigarrow \begin{cases} x = 0 \\ y = 0 \end{cases} \qquad P_0 = (0,0) \quad f(P_0) = 0$$

3) PUNTI SUL BORDO $\rightsquigarrow$ MOLTIPLICATORI DI LAGRANGE

BORDO DI A: $\Phi(x,y) = x^2+2y^2-5 = 0$

$$\text{SISTEMA 1} \begin{cases} \Phi_x = 2x = 0 \\ \Phi_y = 5y = 0 \\ \Phi = 0 \end{cases} \rightsquigarrow \begin{cases} x = 0 \\ y = 0 \\ -5 = 0 \end{cases}$$

NESSUNA SOLUZIONE

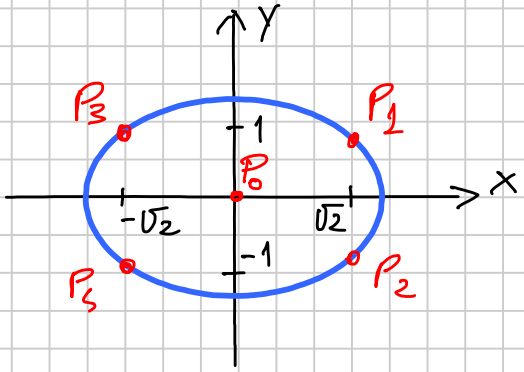
$$\text{SISTEMA 2} \begin{cases} f_x = \lambda \Phi_x \\ f_y = \lambda \Phi_y \\ \Phi = 0 \end{cases} \rightsquigarrow \begin{cases} y = \lambda \cdot 2x \\ x = \lambda \cdot 5y \\ x^2+2y^2-5 = 0 \end{cases} \rightsquigarrow$$

$$\rightsquigarrow \begin{cases} 2y \cdot y = 2y \cdot \lambda \cdot 2x = 5\lambda 2xy \\ x \cdot x = x \cdot \lambda \cdot 5y = 5\lambda xy \end{cases} \rightsquigarrow x^2 = 2y^2$$

$\Phi = 0 \rightsquigarrow x^2+x^2-5 = 0 \quad 2x^2 = 5 \quad x = \pm\sqrt{2} \rightsquigarrow y = \pm 1$

$\rightsquigarrow P_1 = (\sqrt{2}, 1)$, $P_2 = (\sqrt{2}, -1)$, $P_3 = (-\sqrt{2}, 1)$, $P_5 = (-\sqrt{2}, -1)$

$$\begin{cases} f(P_1) = f(P_4) = \sqrt{2} \\ f(P_2) = f(P_3) = -\sqrt{2} \end{cases}$$



$$\rightsquigarrow \begin{cases} MAX = \sqrt{2} \quad IN \quad P_1, P_4 \\ MIN = -\sqrt{2} \quad IN \quad P_2, P_3 \end{cases}$$