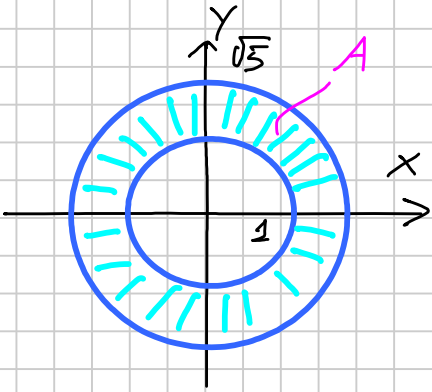
$$f(x,y)=x^2y+y^2 \qquad A:(x^2+y^2-3)^2\le 4$$

$$-2\le x^2+y^2-3\le 2$$

$$\begin{cases} x^2+y^2\ge 1 \\ x^2+y^2\le 5 \end{cases}$$



A COMPATTO $\rightsquigarrow \exists$ MAX E MIN

1) P.TI SINGOLARI INTERNI

$\rightsquigarrow$ NON PRESENTI

2) P.TI STAZIONARI INTERNI

$$\begin{cases} f_x=2xy=0 \\ f_y=x^2+2y=0 \end{cases} \quad \begin{cases} x=0 \\ y=0 \end{cases} \notin A$$

3) BORDO ESTERNO $\Phi(x,y)=x^2+y^2-5=0$

SISTEMA 1

$$\begin{cases} \Phi_x=0 \\ \Phi_y=0 \\ \Phi=0 \end{cases} \rightsquigarrow \begin{cases} 2x=0 \\ 2y=0 \\ x^2+y^2-5=0 \end{cases} \quad \begin{cases} x=0 \\ y=0 \\ -5=0 \end{cases}$$

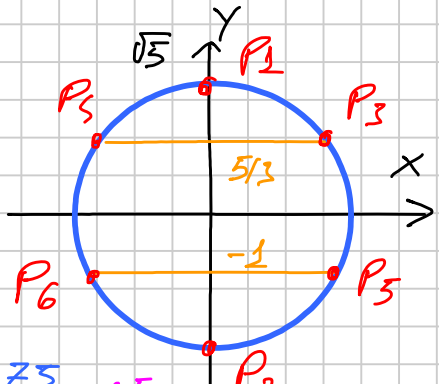
NESSUNA SOLUZIONE

SISTEMA 2

$$\begin{cases} f_x=\lambda\Phi_x \\ f_y=\lambda\Phi_y \\ \Phi=0 \end{cases} \rightsquigarrow \begin{cases} 2xy=2\lambda x \\ x^2+2y=2\lambda y \\ x^2+y^2-5=0 \end{cases} \qquad 2x(\lambda-y)=0$$

$$\begin{cases} x=0 \rightsquigarrow y=\pm\sqrt{5} \rightsquigarrow P_1, P_2 \\ \lambda=y \rightsquigarrow \begin{cases} x^2+2y=2y^2 \\ x^2=5-y^2 \end{cases} \rightsquigarrow 3y^2-2y-5=0 \end{cases}$$

$$y=\frac{2\pm\sqrt{5+60}}{6}=\frac{2\pm 8}{6} \;\begin{matrix} \nearrow \frac{5}{3} \rightsquigarrow P_3, P_4 \\ \searrow -1 \rightsquigarrow P_5, P_6 \end{matrix}$$



$$\begin{cases} f(P_1)=f(P_2)=5 \\ f(P_3)=f(P_4)=\left(5-\frac{25}{9}\right)\frac{5}{3}+\frac{25}{9}=\frac{100}{27}+\frac{75}{27}=\frac{175}{27}\simeq 6{,}5 \\ f(P_5)=f(P_6)=(5-1)(-1)+1=-5+1=-3 \end{cases}$$

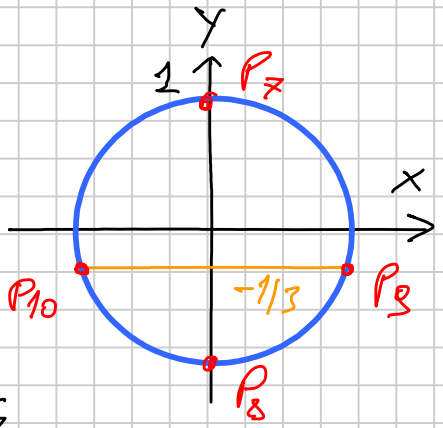
5) BORDO INTERNO $\Phi(x,y) = x^2+y^2-1 = 0$

SISTEMA 1
$$\begin{cases} \Phi_x = 0 \\ \Phi_y = 0 \\ \Phi = 0 \end{cases} \rightsquigarrow \begin{cases} 2x = 0 \\ 2y = 0 \\ x^2+y^2-1 = 0 \end{cases} \quad \begin{cases} x = 0 \\ y = 0 \\ -1 = 0 \end{cases}$$
NESSUNA SOLUZIONE

SISTEMA 2
$$\begin{cases} f_x = \lambda \Phi_x \\ f_y = \lambda \Phi_y \\ \Phi = 0 \end{cases} \rightsquigarrow \begin{cases} 2xy = 2\lambda x \\ x^2+2y = 2\lambda y \\ x^2+y^2-1 = 0 \end{cases} \qquad 2x(\lambda - y) = 0$$

$$\begin{cases} x = 0 \rightsquigarrow y = \pm 1 \rightsquigarrow P_7, P_8 \\ \lambda = y \rightsquigarrow \begin{cases} x^2+2y = 2y^2 \\ x^2 = 1-y^2 \end{cases} \rightsquigarrow 3y^2-2y-1 = 0 \end{cases}$$

$$y = \frac{2 \pm \sqrt{5+12}}{6} = \frac{2 \pm 5}{6} \begin{cases} 1 \rightsquigarrow P_7 \\ -1/3 \rightsquigarrow P_9, P_{10} \end{cases}$$



$$\begin{cases} f(P_7) = f(P_8) = 1 \\ f(P_9) = f(P_{10}) = \left(1-\frac{1}{9}\right)\left(-\frac{1}{3}\right) + \frac{1}{9} = \frac{-8}{27} + \frac{3}{27} = \frac{-5}{27} \end{cases}$$

$$\rightsquigarrow \begin{cases} \text{MAX} \quad 175/27 \quad \text{IN} \quad P_3 \text{ E } P_5 \\ \text{MIN} \quad -3 \quad \text{IN} \quad P_5 \text{ E } P_6 \end{cases}$$