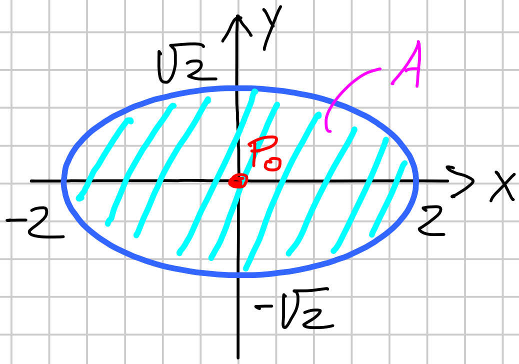
BY GIMUSI 16/04/2014

$f(x,y) = x^2 + y^2 - xy$ $\qquad$ $A: \ x^2 + 2y^2 \leq 4$

A É COMPATTO $\rightsquigarrow \exists$ MAX, MIN



1) PUNTI SING. INTERNI

$\rightsquigarrow$ NON CI SONO

2) PUNTI STAZIONARI INTERNI

$$\begin{cases} f_x = 2x - y = 0 \\ f_y = 2y - x = 0 \end{cases} \rightsquigarrow \begin{cases} x = 0 \\ y = 0 \end{cases} \qquad P_0 = (0,0) \quad f(0,0) = 0$$

3) PUNTI SUL BORDO $\rightsquigarrow$ MOLTIPLICATORI DI LAGRANGE

BORDO DI A: $\Phi(x,y) = x^2 + 2y^2 - 4 = 0$

SISTEMA 1 $\begin{cases} \Phi_x = 2x = 0 \\ \Phi_y = 4y = 0 \\ \Phi = x^2 + 2y^2 - 4 = 0 \end{cases} \rightsquigarrow \begin{cases} x = 0 \\ y = 0 \\ -4 = 0 \end{cases}$ NESSUNA SOLUZIONE

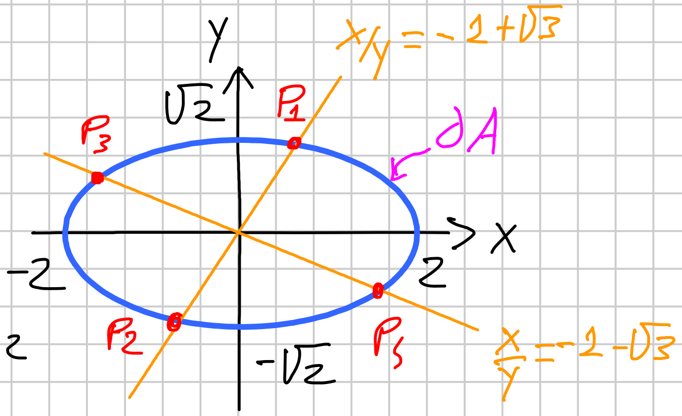
SISTEMA 2 $\begin{cases} f_x = \lambda \Phi_x \\ f_y = \lambda \Phi_y \\ \Phi = 0 \end{cases} \rightsquigarrow \begin{cases} 2x - y = 2\lambda x \\ 2y - x = 4\lambda y \\ x^2 + 2y^2 - 4 = 0 \end{cases} \rightsquigarrow$

$$\rightsquigarrow \quad \frac{2x - y}{2y - x} = \frac{x}{2y} \qquad (2x - y) \cdot 2y = x \cdot (2y - x)$$

$$4xy - 2y^2 - 2xy + x^2 = 0 \qquad x^2 + 2xy - 2y^2 = 0$$

$$\left(\frac{x}{y}\right)^2+2\frac{x}{y}-2=0 \qquad \frac{x}{y}=\frac{-2\pm\sqrt{4+8}}{2}$$

$$\frac{x}{y}=-1\pm\sqrt3$$



PUNTI P_1 E P_2 $\quad \frac{x}{y}=-1+\sqrt3$

$$x^2=(1+3-2\sqrt3)\,y^2=(4-2\sqrt3)\,y^2$$

$$\Phi=x^2+2y^2-4=(4-2\sqrt3)y^2+2y^2-4=0 \qquad y^2=\frac{4}{6-2\sqrt3}$$

$$y^2=\frac{2}{3-\sqrt3}\,\frac{3+\sqrt3}{3+\sqrt3}=\frac{6+2\sqrt3}{9-3}=1+\frac{\sqrt3}{3}$$

$$xy=(-1+\sqrt3)\,y^2=(-1+\sqrt3)\left(1+\frac{\sqrt3}{3}\right)=\cancel{-1}+\sqrt3-\frac{\sqrt3}{3}+\cancel{1}=\frac{2\sqrt3}{3}$$

$$f(x,y)\Big|_{P_1,P_2}=x^2+y^2-xy=(4-2\sqrt3)y^2+y^2-\frac{2\sqrt3}{3}=$$

$$=(5-2\sqrt3)\left(1+\frac{\sqrt3}{3}\right)-2\frac{\sqrt3}{3}=5+\frac{5}{3}\sqrt3-2\sqrt3-2-\frac{2}{3}\sqrt3$$

$$=3-\sqrt3\simeq1{,}26$$

PUNTI P_3 E P_4 $\quad \frac{x}{y}=-1-\sqrt3 \qquad x^2=(4+2\sqrt3)\,y^2$

$$\Phi=x^2+2y^2-4=(6+2\sqrt3)y^2-4=0 \qquad y^2=\frac{2}{3+\sqrt3}\,\frac{3-\sqrt3}{3-\sqrt3}$$

$$y^2=1-\frac{\sqrt3}{3} \qquad xy=(-1-\sqrt3)\left(1-\frac{\sqrt3}{3}\right)=\cancel{-1}+\frac{\sqrt3}{3}-\sqrt3\,\cancel{+1}=-\frac{2}{3}\sqrt3$$

$$f(x,y)\Big|_{P_3,P_4}=x^2+y^2-xy=(4+2\sqrt3)y^2+y^2+\frac{2}{3}\sqrt3=$$

$$=(5+2\sqrt3)\left(1-\frac{\sqrt3}{3}\right)+\frac{2}{3}\sqrt3=5-5\frac{\sqrt3}{3}+2\sqrt3-2+\frac{2}{3}\sqrt3=3+\sqrt3$$

$\simeq 4{,}7$

$$\leadsto \begin{cases} \text{MAX IN } P_3 \text{ E } P_4 = 3+\sqrt3 \\ \text{MIN IN } P_0 = 0 \end{cases}$$