Isometrie dello spazio 1

Argomenti: isometrie dello spazio **Difficoltà:** ⋆⋆⋆⋆

Prerequisiti: isometrie nel piano e nello spazio, matrici ortogonali

Nel seguito sono descritte alcune isometrie dello spazio. Per ciascuna di esse si richiede di

(a) • scrivere l'espressione generale,

(b) • determinare i punti fissi,

(c) • determinare l'immagine del piano $2x + 3y + 5z + 7 = 0$ e della retta $(1, 2, 3) + t(-1, 2, 1)$,

(d) • determinare quale piano va a finire nel piano $2x+3y+5z+7 = 0$ e quale retta va a finire nella retta $(1, 2, 3) + t(-1, 2, 1)$,

(e) • determinare l'immagine della sfera di equazione $x^2 + y^2 + z^2 + 3x - 2y = 10$.

Isometrie da esaminare:

(1) traslazione di vettore $(3, -1, 5)$,

(2) simmetria rispetto al piano xy,

(3) simmetria rispetto al piano yz,

(4) simmetria rispetto al piano $z = 3$,

(5) simmetria rispetto al piano $x = -5$,

(6) simmetria rispetto al piano $y = 3$ seguita da simmetria rispetto al piano $z = -2$,

(7) simmetria rispetto al piano $z = -2$ seguita da simmetria rispetto al piano $y = 3$,

(8) simmetria rispetto al piano $z = -2$ seguita da simmetria rispetto al piano $y = 3$, seguita a sua volta da simmetria rispetto al piano $x = 1$,

(9) simmetria centrale rispetto al punto $(-1, 3, 4)$,

(10) simmetria rispetto al piano $x + 2y - 3z = 0$,

(11) simmetria rispetto al piano $x + 2y - 3z = 5$,

(12) rotazione intorno all'asse z di 90° in un verso giudicato orario da un omino orientato secondo il semiasse positivo delle z,

(13) rotazione intorno all'asse y di 90° in un verso giudicato antiorario da un omino orientato secondo il semiasse positivo delle y,

(14) rotazione intorno all'asse x di 45°, in un verso giudicato antiorario da un omino orientato secondo il semiasse positivo delle x, seguita da simmetria rispetto al piano $x = 3$,

(15) rotazione, intorno alla retta passante per $(1, 2, 3)$ e parallela all'asse x, di 30° in un verso giudicato orario da un omino orientato secondo il semiasse positivo delle x.

SOLUZIONI ?
BY GIMUSI 11/02/15

(1) traslazione di vettore $(3, -1, 5)$,

(a) $P' = P + b \qquad b = (3,-1,5)$

(b) $P' = P + b = P \rightsquigarrow b = 0 \rightsquigarrow$ NO PUNTI FISSI

(c) $\sigma : 2x+3y+5z+7=0 \qquad P=(-1,0,-1) \in \sigma$

$\sigma' : 2x+3y+5z+d'=0 \qquad P' = P+b = (2,-1,4)$

$P' \in \sigma' \rightsquigarrow 4-3+20+d'=0 \quad d'=-21 \rightsquigarrow 2x+3y+5z-21=0$

$r : (1,2,3)+\delta(-1,2,1) = P_0 + \delta V \qquad P_0' = (4,1,8)$

$r' : P_0' + \delta V = (4,1,8)+\delta(-1,2,1)$

(d) $P' = P + b \rightsquigarrow P = P' - b$

$\sigma' : 2x+3y+5z+7=0 \qquad P' = (-1,0,-1) \in \sigma'$

$\sigma : 2x+3y+5z+d=0 \qquad P = (-4,1,-6)$

$P \in \sigma \rightsquigarrow -8+3-30+d=0 \quad d=35 \rightsquigarrow 2x+3y+5z+35=0$

$r' : (1,2,3)+\delta(-1,2,1) = P_0' + \delta V \qquad P_0 = (-2,3,-2)$

$r : P_0 + \delta V = (-2,3,-2)+\delta(-1,2,1)$

(e) $c : x^2+y^2+z^2+3x-2y=10$

$(x-a)^2+(y-b)^2+(z-c)^2 = R^2 \rightsquigarrow x^2+y^2+z^2-2ax-2by-2cz = R^2-a^2-b^2-c^2$

CENTRO $C = (\overset{a}{-\frac{3}{2}}, \overset{b}{1}, \overset{c}{0}) \qquad R^2 = 10 + \overset{a^2}{\frac{9}{4}} + \overset{b^2}{1} = \frac{53}{4} \qquad R = \frac{\sqrt{53}}{2}$

$C' = C + b = (-\frac{3}{2}+3, 1-1, 0+5) = (\frac{3}{2}, 0, 5) \qquad R' = R$

$c' : (x-\frac{3}{2})^2 + y^2 + (z-5)^2 = \frac{53}{4} \qquad x^2+y^2+z^2-3x-10z = \frac{53}{4} - \frac{9}{4} - 25 = -\frac{56}{4}$

$x^2+y^2+z^2-\frac{3}{2}x-10z = -14$

(2) simmetria rispetto al piano xy,

(a) $P' = SP \qquad S = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$

(b) $P' = SP = P \rightsquigarrow (S-I)P = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -2 \end{pmatrix} P = 0 \quad P = \sigma \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + s \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

$\rightsquigarrow$ PUNTI FISSI $\equiv$ PIANO XY

(c) $\pi: 2x+3y+5z+7=0 \qquad n=(2,3,5) \perp \pi \qquad P=(-1,0,-1) \in \pi$

$n' = Sn = (2,3,-5) \quad P' = SP = (-1,0,1) \quad \pi': 2x+3y-5z+d'=0$

$P' \in \pi' \rightsquigarrow -2-5+d'=0 \quad d'=7 \rightsquigarrow 2x+3y-5z+7=0$

$r: (1,2,3)+\sigma(-1,2,1) = P_0 + \sigma V \qquad P_0' = (1,2,-3) \quad V' = (-1,2,-1)$

$r': \ P_0' + \sigma V' = (1,2,-3)+\sigma(-1,2,-1)$

(d) $P' = SP \rightsquigarrow P = S^{-1}P' = S^{\delta}P' = SP'$

$\pi': 2x+3y+5z+7=0 \rightsquigarrow \pi: 2x+3y-5z+7=0$

$r': (1,2,3)+\sigma(-1,2,1) \rightsquigarrow r: (1,2,-3)+\sigma(-1,2,-1)$

(e) $C: x^2+y^2+z^2+3x-2y=10$

$(x-a)^2+(y-b)^2+(z-c)^2=R^2 \rightsquigarrow x^2+y^2+z^2-2ax-2by-2cz=R^2-a^2-b^2-c^2$

CENTRO $C = (\overset{a}{-\frac{3}{2}}, \overset{b}{1}, \overset{c}{0}) \qquad R^2 = 10 + \overset{a^2}{\frac{9}{4}} + \overset{b^2}{1} = \frac{53}{4} \qquad R = \frac{\sqrt{53}}{2}$

$C' = SC = (-\frac{3}{2}, 1, 0) \equiv C \qquad R' = R$

$C': \ x^2+y^2+z^2+3x-2y=10$

(3) simmetria rispetto al piano yz,

(a) $P' = SP \quad S = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

(b) $P' = SP = P \rightsquigarrow (S-I)P = \begin{pmatrix} -2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} P = 0 \quad P = t\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + s\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

$\rightsquigarrow$ PUNTI FISSI $\equiv$ PIANO yz

(c) $\pi : 2x+3y+5z+7=0 \qquad n=(2,3,5)\perp \pi \qquad P=(-1,0,-1)\in\pi$

$n' = Sn = (-2,3,5) \quad P' = SP = (1,0,-1) \quad \pi' : -2x+3y+5z+d'=0$

$P' \in \pi' \rightsquigarrow -2-5+d'=0 \quad d'=7 \rightsquigarrow -2x+3y+5z+7=0$

$r : (1,2,3)+t(-1,2,1) = P_0 + tV \qquad P_0' = (-1,2,3) \quad V' = (1,2,1)$

$r' : P_0' + tV' = (-1,2,3)+t(1,2,1)$

(d) $P' = SP \rightsquigarrow P = S^{-1}P' = S^{\delta}P' = SP'$

$\pi' : 2x+3y+5z+7=0 \rightsquigarrow \pi : -2x+3y+5z+7=0$

$r' : (1,2,3)+t(-1,2,1) \rightsquigarrow r : (-1,2,3)+t(1,2,1)$

(e) $C : x^2+y^2+z^2+3x-2y=10$

$(x-a)^2+(y-b)^2+(z-c)^2=R^2 \rightsquigarrow x^2+y^2+z^2-2ax-2by-2cz=R^2-a^2-b^2-c^2$

CENTRO $C = (\overset{a}{-\frac{3}{2}}, \overset{b}{1}, \overset{c}{0}) \qquad R^2 = 10 + \overset{a^2}{\frac{9}{4}} + \overset{b^2}{1} = \frac{53}{4} \qquad R = \frac{\sqrt{53}}{2}$

$C' = SC = (\frac{3}{2}, 1, 0) \qquad R' = R$

$C' : (x-\frac{3}{2})^2+(y-1)^2+z^2 = \frac{53}{4} \quad x^2+y^2+z^2-3x-2y = \frac{53}{4}-\frac{9}{4}-1 = \frac{40}{4}$

$x^2+y^2+z^2-3x-2y=10$

(4) simmetria rispetto al piano $z = 3$,

(a) $P' = S(P-A)+A \quad A \in \delta : z=3 \qquad S = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$

(b) $P' = S(P-A)+A = P \quad (S-I)(P-A) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -2 \end{pmatrix}(P-A) = 0 \quad P-A = \delta \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + s \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

$\rightsquigarrow P = (\delta, s, 3) \equiv$ PIANO $z=3$

(c) $\delta : 2x+3y+5z+7=0 \qquad n=(2,3,5) \perp \delta \qquad P=(-1,0,-1) \in \delta$

$n' = (2,3,-5) \quad (P-A) = (-2,0,-5) \quad P' = (-2,0,5)+(0,0,3) = (-2,0,7)$

$\delta' : 2x+3y-5z+d' = 0 \quad P' \in \delta' \rightsquigarrow -2-35+d'=0 \quad d' = 37$

$\rightsquigarrow 2x+3y-5z+37=0$

$r: (1,2,3)+\delta(-1,2,1) = P_0 + \delta V \qquad P_0' = (1,2,3) \quad V' = (-1,2,-1)$

$r' : P_0' + \delta V' = (-1,2,3)+\delta(-1,2,-1)$

(d) $P' = S(P-A)+A \quad S(P-A) = P'-A \quad P = S(P'-A)+A \qquad S^{-1} = S^{\delta} = S$

$\delta' : 2x+3y+5z+7=0 \rightsquigarrow \delta : 2x+3y-5z+37=0$

$r' : (1,2,3)+\delta(-1,2,1) \rightsquigarrow r : (-1,2,3)+\delta(-1,2,-1)$

(e) $C : x^2+y^2+z^2+3x-2y=10$

$(x-a)^2+(y-b)^2+(z-c)^2 = R^2 \rightsquigarrow x^2+y^2+z^2-2ax-2by-2cz = R^2-a^2-b^2-c^2$

CENTRO $C = (\overset{a}{-\frac{3}{2}}, \overset{b}{1}, \overset{c}{0}) \qquad R^2 = 10 + \overset{a^2}{\frac{9}{4}} + \overset{b^2}{1} = \frac{53}{4} \qquad R = \frac{\sqrt{53}}{2}$

$C' = (-\frac{3}{2}, 1, 6) \quad R' = R \qquad C' : (x+\frac{3}{2})^2+(y-1)^2+(z-6)^2 = \frac{53}{4}$

$x^2+y^2+z^2+3x-2y-12z = \frac{53}{4} - \frac{9}{4} - 1 - 36 = \frac{53-9-4-144}{4} = \frac{-104}{4}$

$x^2+y^2+z^2+3x-2y-12z = -26$

(5) simmetria rispetto al piano $x = -5$,

(a) $P' = S(P-A)+A \quad A \in \pi : x=-5 \quad S = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

(b) $P' = S(P-A)+A = P \quad (S-I)(P-A) = \begin{pmatrix} -2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}(P-A) = 0 \quad P-A = \delta \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + s \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

$\leadsto P = (-5, \delta, s) = \text{PIANO } x=-5$

(c) $\pi: 2x+3y+5z+7=0 \quad n=(2,3,5) \perp \pi \quad P=(-1,0,-1) \in \pi$

$n' = (-2,3,5) \quad (P-A) = (4,0,-1) \quad P' = (-4,0,-1)+(-5,0,0) = (-9,0,-1)$

$\pi': -2x+3y+5z+\alpha' = 0 \quad P' \in \pi' \leadsto 18-5+\alpha' = 0 \quad \alpha' = -13$

$\leadsto -2x+3y+5z-13=0$

$r: (1,2,3)+\delta(-1,2,1) = P_0 + \delta v \quad P_0' = (-11,2,3) \quad v' = (1,2,1)$

$r': P_0' + \delta v' = (-11,2,3)+\delta(1,2,-1)$

(d) $P' = S(P-A)+A \quad S(P-A) = P'-A \quad P = S(P'-A)+A \quad S^{-1} = S^{\delta} = S$

$\pi': 2x+3y+5z+7=0 \leadsto \pi: -2x+3y+5z-13=0$

$r': (1,2,3)+\delta(-1,2,1) \leadsto r: (-11,2,3)+\delta(1,2,-1)$

(e) $c: x^2+y^2+z^2+3x-2y=10$

$(x-a)^2+(y-b)^2+(z-c)^2=R^2 \leadsto x^2+y^2+z^2-2ax-2by-2cz=R^2-a^2-b^2-c^2$

CENTRO $C = (\overset{a}{-\frac{3}{2}}, \overset{b}{1}, \overset{c}{0}) \quad R^2 = 10 + \overset{a^2}{\frac{9}{4}} + \overset{b^2}{1} = \frac{53}{4} \quad R = \frac{\sqrt{53}}{2}$

$C' = (-\frac{17}{2}, 1, 0) \quad R' = R \quad c': (x+\frac{17}{2})^2+(y-1)^2+z^2 = \frac{53}{4}$

$x^2+y^2+z^2+17x-2y = \frac{53}{4} - \frac{189}{4} - 1 = \frac{53-189-4}{4}$

$x^2+y^2+z^2+17x-2y = -35$

(6) simmetria rispetto al piano $y = 3$ seguita da simmetria rispetto al piano $z = -2$,

(a) SIMM. σ': $y = 3$ $\quad P' = S'(P-A)+A \quad A \in \sigma' \quad S' = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

SIMM. σ'': $z=-2$ $\quad P'' = S''(P'-B)+B \quad B \in \sigma'' \quad S'' = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$

COMPOSIZIONE: ASSUMIAMO $A = B = (0,3,-2)$

$$P'' = S''[S'(P-A)+A-A]+A = S''S'(P-A)+A$$

$$\leadsto \quad P'' = S(P-A)+A \qquad S = S''S' = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

(b) $P'' = S(P-A)+A = P \leadsto (S-I)(P-A) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -2 \end{pmatrix}(P-A) = 0$

$(P-A) = \delta(1,0,0) \qquad P = (\delta, 3, -2)$ = RETTA // ASSE X PASS. PER A

(c) σ: $2x+3y+5z+7=0 \qquad n=(2,3,5) \perp \sigma \qquad P=(-1,0,-1) \in \sigma$

$n'' = (2,-3,-5) \quad (P-A) = (-1,-3,1) \quad P'' = (-1,3,-1)+(0,3,-2) = (-1,6,-3)$

σ'': $2x-3y-5z+d''=0 \quad P'' \in \sigma'' \leadsto -2-18+15+d''=0 \quad d''=5$

$\leadsto 2x-3y-5z+5=0$

r: $(1,2,3)+\delta(-1,2,1) = P_0 + \delta V \qquad V'' = (-1,-2,-1)$

$(P_0 - A) = (1,-1,5) \quad P_0'' = (1,1,-5)+(0,3,-2) = (1,5,-7)$

r': $P_0'' + \delta V'' = (1,5,-7)+\delta(-1,-2,-1)$

(d) $P'' = S(P-A)+A \qquad S(P-A) = P''-A \qquad P = S(P''-A)+A \qquad S^{-1}=S^T=S$

$\delta'': 2x+3y+3z+7=0 \rightsquigarrow \delta: 2x-3y-5z+5=0$

$r'': (1,2,3)+\sigma(-1,2,1) \rightsquigarrow r: (1,5,-7)+\sigma(-1,-2,-1)$

(e) $C: x^2+y^2+z^2+3x-2y=10$

$(x-a)^2+(y-b)^2+(z-c)^2=R^2 \rightsquigarrow x^2+y^2+z^2-2ax-2by-2cz=R^2-a^2-b^2-c^2$

CENTRO $C=(\overset{a}{-\tfrac{3}{2}},\overset{b}{1},\overset{c}{0}) \qquad R^2=10+\overset{a^2}{\tfrac{9}{4}}+\overset{b^2}{1}=\tfrac{53}{4} \qquad R=\tfrac{\sqrt{53}}{2}$

$(C-A)=(-\tfrac{3}{2},-2,2) \qquad C''=(-\tfrac{3}{2},2,-2)+(0,3,-2)=(-\tfrac{3}{2},5,-4) \qquad R''=R$

$C'': (x+\tfrac{3}{2})^2+(y-5)^2+(z+4)^2=\tfrac{53}{4} \qquad x^2+y^2+z^2+3x-10y+8z=$

$=\tfrac{53}{4}-\tfrac{9}{4}-25-16=\overset{-120}{\tfrac{53-9-100-64}{4}} \qquad x^2+y^2+z^2+3x-10y+8z=-30$

(7) simmetria rispetto al piano $z=-2$ seguita da simmetria rispetto al piano $y=3$,

(a) SIMM. $\delta': z=-2 \qquad P'=S'(P-A)+A \qquad A\in\delta' \qquad S'=\begin{pmatrix}1&0&0\\0&1&0\\0&0&-1\end{pmatrix}$

SIMM. $\delta'': y=3 \qquad P''=S''(P'-B)+B \qquad B\in\delta'' \qquad S''=\begin{pmatrix}1&0&0\\0&-1&0\\0&0&1\end{pmatrix}$

COMPOSIZIONE: ASSUMIAMO $A=B=(0,3,-2)$

$P''=S''[S'(P-A)+A-A]+A=S''S'(P-A)+A$

$\rightsquigarrow P''=S(P-A)+A \qquad S=S''S'=\begin{pmatrix}1&0&0\\0&-1&0\\0&0&-1\end{pmatrix} \equiv$ ES. (6)

(b) (c) (d) (e) $\rightsquigarrow$ Vd. ES.(6)

(8) simmetria rispetto al piano $z=-2$ seguita da simmetria rispetto al piano $y=3$, seguita a sua volta da simmetria rispetto al piano $x=1$,

(a) SIMM. σ': $z=-2$ $\quad P'=S'(P-A)+A \quad A\in\sigma' \quad S'=\begin{pmatrix}1&0&0\\0&1&0\\0&0&-1\end{pmatrix}$

SIMM. σ'': $y=3$ $\quad P''=S''(P'-B)+B \quad B\in\sigma'' \quad S''=\begin{pmatrix}1&0&0\\0&-1&0\\0&0&1\end{pmatrix}$

SIMM. σ''': $x=1$ $\quad P'''=S'''(P''-C)+C \quad C\in\sigma''' \quad S'''=\begin{pmatrix}-1&0&0\\0&1&0\\0&0&1\end{pmatrix}$

COMPOSIZIONE: ASSUMIAMO $A=B=C=(1,3,-2)$

$$P'''=S'''\{S''[S'(P-A)\cancel{+A}\cancel{-A}]\cancel{+A}\cancel{-A}\}+A=S'''S''S'(P-A)+A$$

$\rightsquigarrow P'''=S(P-A)+A \qquad S=S'''S''S'=\begin{pmatrix}-1&0&0\\0&-1&0\\0&0&-1\end{pmatrix}$

(b) $P'''=S(P-A)+A=P \rightsquigarrow (S-I)(P-A)=\begin{pmatrix}-2&0&0\\0&-2&0\\0&0&-2\end{pmatrix}(P-A)=0$

$P-A=0 \rightsquigarrow P=A$ $\equiv$ 1 P.TO FISSO $\equiv$ SIMM. CENTRALE P.TO A

(c) $\sigma: 2x+3y+5z+7=0 \qquad n=(2,3,5)\perp\sigma \qquad P=(-1,0,-1)\in\sigma$

$n'''=(-2,-3,-5) \quad (P-A)=(-2,-3,1) \quad P'''=(2,3,-1)+(1,3,-2)=(3,6,-3)$

$\sigma''': -2x-3y-5z+d'''=0 \quad P'''\in\sigma''' \rightsquigarrow -6-18+15+d'''=0 \quad d'''=9$

$\rightsquigarrow -2x-3y-5z+9=0$

$r: (1,2,3)+\delta(-1,2,1)=P_0+\delta V \qquad V'''=(1,-2,-1)$

$(P_0-A)=(0,-1,5) \quad P_0'''=(0,1,-5)+(1,3,-2)=(1,4,-7)$

$r': P_0'''+\delta V'''=(1,4,-7)+\delta(1,-2,-1)$

(d) $P''' = S(P-A)+A \quad S(P-A) = P'''-A \quad P = S(P'''-A)+A \quad S^{-1} = S^{T} = S$

$\pi''': 2x+3y+5z+7=0 \leadsto \pi: -2x-3y-5z+9=0$

$r''': (1,2,3)+t(-1,2,1) \leadsto r: (1,4,-7)+t(1,-2,-1)$

(e) $C: x^2+y^2+z^2+3x-2y=10$

$(x-a)^2+(y-b)^2+(z-c)^2=R^2 \leadsto x^2+y^2+z^2-2ax-2by-2cz=R^2-a^2-b^2-c^2$

CENTRO $C = (\overset{a}{-\frac{3}{2}}, \overset{b}{1}, \overset{c}{0}) \quad R^2 = 10+\overset{a^2}{\frac{9}{4}}+\overset{b^2}{1} = \frac{53}{4} \quad R = \frac{\sqrt{53}}{2}$

$(C-A) = (-\frac{5}{2},-2,2) \quad C''' = (\frac{5}{2},2,-2)+(1,3,-2) = (\frac{7}{2},5,-4) \quad R''' = R$

$C''': (x-\frac{7}{2})^2+(y-5)^2+(z+4)^2 = \frac{53}{4} \quad x^2+y^2+z^2-7x-10y+8z =$

$= \frac{53}{4}-\frac{49}{4}-25-16 = \frac{\overset{-160}{53-49-100-64}}{4} \quad x^2+y^2+z^2-7x-10y+8z=-40$

(9) simmetria centrale rispetto al punto $(-1,3,4)$,

(a) $P' = S(P-A)+A \quad A=(-1,3,4) \quad S = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$

(b) $P=A$ ≡ 1 P.TO FISSO ≡ SIMM. CENTRALE P.TO A

(c) $\pi: 2x+3y+5z+7=0 \quad n=(2,3,5) \perp \pi \quad P=(-1,0,-1) \in \pi$

$n' = (-2,-3,-5) \quad (P-A) = (0,-3,-5) \quad P' = (0,3,5)+(-1,3,4) = (-1,6,9)$

$\pi': -2x-3y-5z+d'=0 \quad P' \in \pi' \leadsto 2-18-45+d'=0 \quad d'=61$

$\leadsto -2x-3y-5z+61=0$

$r: (1,2,3)+t(-1,2,1) = P_0+tv \quad v' = (1,-2,-1)$

$(P_0-A) = (2,-1,-1) \quad P_0' = (-2,1,1)+(-1,3,4) = (-3,4,5)$

$r': P_0'+tv' = (-3,4,5)+t(1,-2,-1)$

(d) $P' = S(P-A)+A \qquad S(P-A) = P'-A \qquad P = S(P'-A)+A \qquad S^{-1} = S^{T} = S$

$\pi' : 2x+3y+5z+7=0 \rightsquigarrow \pi : -2x-3y-5z+61=0$

$r' : (1,2,3)+t(-1,2,1) \rightsquigarrow r : (-3,4,5)+t(1,-2,-1)$

(e) $C : x^2+y^2+z^2+3x-2y=10$

$(x-a)^2+(y-b)^2+(z-c)^2=R^2 \rightsquigarrow x^2+y^2+z^2-2ax-2by-2cz=R^2-a^2-b^2-c^2$

CENTRO $C=(\overset{a}{-\tfrac{3}{2}},\overset{b}{1},\overset{c}{0}) \qquad R^2=10+\overset{a^2}{\tfrac{9}{4}}+\overset{b^2}{1}=\tfrac{53}{4} \qquad R=\tfrac{\sqrt{53}}{2}$

$(C-A)=(-\tfrac{1}{2},-2,-4) \qquad C'=(\tfrac{1}{2},2,4)+(-1,3,4)=(-\tfrac{1}{2},5,8) \qquad R'=R$

$C' : (x+\tfrac{1}{2})^2+(y-5)^2+(z-8)^2=\tfrac{53}{4} \qquad x^2+y^2+z^2+x-10y-16z =$

$= \tfrac{53}{4}-\tfrac{1}{4}-25-64 = \dfrac{\overset{-304}{53-1-100-256}}{4} \qquad x^2+y^2+z^2+x-10y-16z=-76$

(10) simmetria rispetto al piano $x+2y-3z=0$,

(a) $v_1, v_2 \in \pi : \quad v_1=(-2,1,0) \qquad v_2(3,0,1)$

$v_3 \perp \pi : \quad v_3 = \begin{vmatrix} e_1 & e_2 & e_3 \\ -2 & 1 & 0 \\ 3 & 0 & 1 \end{vmatrix} = (1,2,-3)$

NELLA BASE $\{v_1,v_2,v_3\}$ $\qquad \hat{S} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$

MATRICE DI CAMBIO BASE $\qquad M = \begin{pmatrix} -2 & 3 & 1 \\ 1 & 0 & 2 \\ 0 & 1 & -3 \end{pmatrix} \rightsquigarrow$ DALLA VECCHIA ALLA CANONICA

$DET(M)=1+4+9=14 \qquad M^{-1} = \frac{1}{14}\begin{pmatrix} -2 & +3 & 1 \\ +10 & 6 & +2 \\ 6 & +5 & -3 \end{pmatrix}^{T} = \frac{1}{14}\begin{pmatrix} -2 & 10 & 6 \\ 3 & 6 & 5 \\ 1 & 2 & -3 \end{pmatrix}$

IN BASE CANONICA: $S = M\hat{S}M^{-1} = \frac{1}{15} M \begin{pmatrix} -2 & 10 & 6 \\ 3 & 6 & 5 \\ -1 & -2 & 3 \end{pmatrix} =$

$(S S^{\delta} = I \rightsquigarrow$ ISOMETRIA$)$

$\rightsquigarrow S = \frac{1}{15} \begin{pmatrix} 12 & -5 & 6 \\ -5 & 6 & 12 \\ 6 & 12 & -5 \end{pmatrix}$ $\quad P' = SP$

(b) $P' = SP = P \rightsquigarrow (S-I)P = \frac{1}{15} \begin{pmatrix} -2 & -5 & 6 \\ -5 & -8 & 12 \\ 6 & 12 & -18 \end{pmatrix} P = 0 \rightsquigarrow$

$\rightsquigarrow -2x-5y+6z=0 \equiv x+2y-3z=0$ $\equiv$ PIANO FISSO DI SIMM.

(c) $\pi: 2x+3y+5z+7=0 \quad m=(2,3,5) \perp \pi \quad P=(-1,0,-1) \in \pi$

$m' = Sm = \frac{1}{15}(52, 70, 20) = (3,5,2) \quad P' = SP = \frac{1}{15}(-18,-8,-2)$

$P' = \left(-\frac{8}{7}, \frac{-5}{7}, \frac{-1}{7}\right) \quad \pi': 3x+5y+2z+d' \quad P' \in \pi' \rightsquigarrow -\frac{27}{7} - \frac{20}{7} - \frac{2}{7} + d' = 0$

$d' = -\frac{59}{7} \rightsquigarrow 3x+5y+2z-7=0$

$r: (1,2,3)+t(-1,2,1) = P_0 + tV \qquad v' = (-1,2,1) = V \in \pi$

$P_0' = \frac{1}{15}(22,55,18) = \left(\frac{11}{7}, \frac{22}{7}, \frac{9}{7}\right) \quad r': \left(\frac{11}{7}, \frac{22}{7}, \frac{9}{7}\right) + t(-1,2,1)$

(d) $P' = SP \rightsquigarrow P = S^{-1}P' = S^{\delta}P' = SP'$ $\quad S^{-1} = S^{\delta} = S$

$\pi': 2x+3y+5z+7=0 \rightsquigarrow \pi: 3x+5y+2z-7=0$

$r': (1,2,3)+t(-1,2,1) \rightsquigarrow r: \left(\frac{11}{7}, \frac{22}{7}, \frac{9}{7}\right) + t(-1,2,1)$

(e) $c: x^2+y^2+z^2+3x-2y=10$ $\quad x^2+y^2+z^2-2ax-2by-2cz = R^2-a^2-b^2-c^2$

CENTRO $C = \left(\overset{a}{-\frac{3}{2}}, \overset{b}{1}, \overset{c}{0}\right) \quad R^2 = 10 + \overset{a^2}{\frac{9}{5}} + \overset{b^2}{1} = \frac{53}{5} \qquad R = \frac{\sqrt{53}}{2}$

$C' = \frac{1}{15}(-22,12,3) = \left(-\frac{11}{7}, \frac{6}{7}, \frac{3}{15}\right) \quad c': \left(x+\frac{11}{7}\right)^2 + \left(y-\frac{6}{7}\right)^2 + \left(z-\frac{3}{15}\right)^2 = \frac{53}{5}$

$x^2+y^2+z^2+\frac{22}{7}x-\frac{12}{7}y-\frac{3}{7}z = \frac{53}{5} - \frac{121}{59} - \frac{36}{59} - \frac{9}{196} = \frac{2597-585-155-9}{196} = \frac{1960}{196}$

$x^2+y^2+z^2+\frac{22}{7}x-\frac{12}{7}y-\frac{3}{7}z = 10$

(11) simmetria rispetto al piano $x + 2y - 3z = 5$,

Vd. ES. (10)

(a) $P' = S(P-A)+A \qquad A=(1,2,0)\in\pi \qquad S = \frac{1}{14}\begin{pmatrix} 12 & -4 & 6 \\ -4 & 6 & 12 \\ 6 & 12 & -4 \end{pmatrix}$

(b) $x+2y-3z=5$ = PIANO FISSO DI SIMM.

(c) $\pi: 2x+3y+5z+7=0 \qquad n=(2,3,5)\perp\pi \qquad P=(-1,0,-1)\in\pi$

$n' = Sn = \frac{1}{14}(42,70,28)=(3,5,2) \qquad (P-A)=(-2,-2,-1)$

$S(P-A) = \frac{1}{14}(-22,-16,-32) = \left(-\frac{11}{7}, -\frac{8}{7}, -\frac{16}{7}\right) \qquad P' = \left(-\frac{4}{7}, \frac{6}{7}, -\frac{16}{7}\right)$

$\pi': 3x+5y+2z+d'=0 \qquad P'\in\pi' \rightsquigarrow -\frac{12}{7}+\frac{30}{7}-\frac{32}{7}+d'=0 \qquad d'=2$

$\rightsquigarrow 3x+5y+2z+2=0$

$r: (1,2,3)+\sigma(-1,2,1) = P_0+\sigma V \qquad V'=(-1,2,1) = V \,//\, \pi$

$(P_0-A)=(0,0,3) \qquad S(P_0-A)=\frac{1}{14}(18,36,-12)=\left(-\frac{9}{7},\frac{18}{7},\frac{-6}{7}\right)$

$P_0' = \left(-\frac{2}{7},\frac{32}{7},\frac{-6}{7}\right) \qquad r': \left(-\frac{2}{7},\frac{32}{7},\frac{-6}{7}\right)+\sigma(-1,2,1)$

(d) $P' = S(P-A)+A \rightsquigarrow P = S(P'-A)+A \qquad S^{-1}=S^{T}=S$

$\pi': 2x+3y+5z+7=0 \rightsquigarrow \pi: 3x+5y+2z+2=0$

$r': (1,2,3)+\sigma(-1,2,1) \rightsquigarrow r: \left(-\frac{2}{7},\frac{32}{7},\frac{-6}{7}\right)+\sigma(-1,2,1)$

(e) $c: x^2+y^2+z^2+3x-2y=10 \qquad x^2+y^2+z^2-2ax-2by-2cz=R^2-a^2-b^2-c^2$

CENTRO $C=(\overset{a}{-\tfrac{3}{2}},\overset{b}{1},\overset{c}{0}) \qquad R^2=10+\overset{a^2}{\tfrac{9}{4}}+\overset{b^2}{1}=\frac{53}{4} \qquad R=\frac{\sqrt{53}}{2}$

$(C-A)=\left(-\frac{5}{2},-1,0\right) \qquad S(C-A)=\frac{1}{14}(-26,4,-27)=\left(-\frac{13}{7},\frac{2}{7},\frac{-27}{14}\right)$

$C'=\left(\frac{-6}{7},\frac{16}{7},\frac{-27}{14}\right) \qquad c': x^2+y^2+z^2+\frac{12}{7}x-\frac{32}{7}y+\frac{27}{14}z=\frac{53}{4}-\frac{36}{49}-\frac{256}{49}-\frac{729}{196}$

$=\frac{2597-144-1024-729}{196}=\frac{160}{49} \qquad x^2+y^2+z^2+\frac{12}{7}x-\frac{32}{7}y+\frac{27}{14}z=\frac{160}{49}$

(12) rotazione intorno all'asse z di $90°$ in un verso giudicato orario da un omino orientato secondo il semiasse positivo delle z,

(a) $P' = SP$ $\quad S = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \xrightarrow{\theta = -\frac{\pi}{2}}$ $\quad S = \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

(b) $P' = SP = P \rightsquigarrow (S - I)P = \begin{pmatrix} -1 & 1 & 0 \\ -1 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} = 0$ $\quad P = (0, 0, t) \equiv$ ASSE z

(c) $\pi: 2x + 3y + 5z + 7 = 0$ $\quad n = (2, 3, 5) \perp \pi$ $\quad P = (-1, 0, -1) \in \pi$

$n' = (3, -2, 5)$ $\quad P' = (0, 1, -1)$ $\quad \pi': 3x - 2y + 5z + d' = 0$

$P' \in \pi' \rightarrow -2 - 5 + d'$ $\quad d' = 7$ $\rightsquigarrow$ $3x - 2y + 5z + 7 = 0$

$r: (1\ 2, 3) + t(-1, 2, 1) = P_0 + tV$ $\quad V' = (2, 1, 1)$ $\quad P_0' = (2, -1, 3)$

$r': (2, -1, 3) + t(2, 1, 1)$

(d) $P' = SP \rightsquigarrow P = S^{-1}P' = S^T P'$

$\pi': 2x + 3y + 5z + 7 = 0$ $\quad n = (-3, 2, 5)$ $\quad P = (0, -1, -1)$

$\pi: -3x + 2y + 5z + d' = 0$ $\quad P \in \pi \rightsquigarrow -2 - 5 + d'$ $\quad d' = 7$

$-3x + 2y + 5z + 7 = 0$

$r': (1, 2, 3) + t(-1, 2, 1) \rightsquigarrow r: (-2, 1, 3) + t(-2, -1, 1)$

(e) $c: x^2 + y^2 + z^2 + 3x - 2y = 10$ $\quad x^2 + y^2 + z^2 - 2ax - 2by - 2cz = R^2 - a^2 - b^2 - c^2$

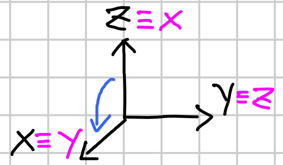
CENTRO $C = (\overset{a}{-\frac{3}{2}}, \overset{b}{1}, \overset{c}{0})$ $\quad R^2 = 10 + \overset{a^2}{\frac{9}{4}} + \overset{b^2}{1} = \frac{53}{4}$ $\quad R = \frac{\sqrt{53}}{2}$

$C' = \left(1, \frac{3}{2}, 0\right)$ $\quad c': x^2 + y^2 + z^2 - 2x - 3y = \frac{53}{4} - 1 - \frac{9}{4} = \frac{40}{4}$

$x^2 + y^2 + z^2 - 2x - 3y = 10$

(13) rotazione intorno all'asse y di 90° in un verso giudicato antiorario da un omino orientato secondo il semiasse positivo delle y,

(a) $P' = SP$



$$S = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{pmatrix} = \overset{M}{\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}} \overset{\hat{S}}{\begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}} \overset{M^{-1}}{\begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}}$$

(b) $P' = SP = P \rightsquigarrow (S-I)P = \begin{pmatrix} -1 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & -1 \end{pmatrix} P = 0$ $\quad P = (0,\sigma,0) \equiv$ ASSE Y

(c) $\pi: 2x+3y+5z+7=0 \quad n=(2,3,5)\perp\pi \quad P=(-1,0,-1)\in\pi$

$n'=(5,3,-2) \quad P'=(-1,0,1) \quad \pi': 5x+3y-2z+d'=0$

$P'\in\pi' \rightsquigarrow -5-2+d' \quad d'=7 \quad \rightsquigarrow 5x+3y-2z+7=0$

$r: (1,2,3)+\sigma(-1,2,1) = P_0+\sigma V \quad V'=(1,2,1) \quad P_0'=(3,2,-1)$

$r': (3,2,-1)+\sigma(1,2,1)$

(d) $P'=SP \rightsquigarrow P=S^{-1}P' = S^{3}P'$

$\pi': 2x+3y+5z+7=0 \quad n=(-5,3,2) \quad P=(1,0,-1)$

$\pi: -5x+3y+2z+d'=0 \quad P\in\pi \rightsquigarrow -2-5+d' \quad d'=7$

$-5x+3y+2z+7=0$

$r': (1,2,3)+\sigma(-1,2,1) \rightsquigarrow r: (-3,2,1)+\sigma(-1,2,-1)$

(e) $c: x^2+y^2+z^2+3x-2y=10 \quad x^2+y^2+z^2-2ax-2by-2cz=R^2-a^2-b^2-c^2$

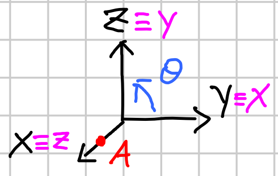
CENTRO $C=(\overset{a}{-\tfrac{3}{2}}, \overset{b}{1}, \overset{c}{0}) \quad R^2=10+\overset{a^2}{\tfrac{9}{4}}+\overset{b^2}{1}=\tfrac{53}{4} \quad R=\tfrac{\sqrt{53}}{2}$

$C'=(0,1,\tfrac{3}{2}) \quad c': x^2+y^2+z^2-2x-3z=\tfrac{53}{4}-1-\tfrac{9}{4}=\tfrac{40}{4}$

$x^2+y^2+z^2-2x-3z=10$

(14) rotazione intorno all'asse x di 45°, in un verso giudicato antiorario da un omino orientato secondo il semiasse positivo delle x, seguita da simmetria rispetto al piano $x=3$,

(a) $P' = S'P$



$$S' = \overset{M}{\begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}} \overset{\hat{S}}{\begin{pmatrix} \sqrt{2}/2 & -\sqrt{2}/2 & 0 \\ \sqrt{2}/2 & \sqrt{2}/2 & 0 \\ 0 & 0 & 1 \end{pmatrix}} \overset{M^{-1}}{\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}} =$$

$$= \begin{pmatrix} 0 & 0 & 1 \\ \sqrt{2}/2 & -\sqrt{2}/2 & 0 \\ \sqrt{2}/2 & \sqrt{2}/2 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \rightsquigarrow S' = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \sqrt{2}/2 & -\sqrt{2}/2 \\ 0 & \sqrt{2}/2 & \sqrt{2}/2 \end{pmatrix}$$

$$P'' = S''(P'-A)+A \qquad A=(3,0,0)\in\sigma: x=3 \qquad S'' = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

COMPOSIZIONE: $P'' = S''[S'P-A]+A = S''S'P - S''A + A$

$$S = S''S' = \begin{pmatrix} -1 & 0 & 0 \\ 0 & \sqrt{2}/2 & -\sqrt{2}/2 \\ 0 & \sqrt{2}/2 & \sqrt{2}/2 \end{pmatrix} \qquad S''A = -A \quad \rightsquigarrow \quad P'' = SP + 2A$$

(b) $$P'' = SP+2A = P \rightsquigarrow (S-I)P = \begin{pmatrix} -2 & 0 & 0 \\ 0 & \sqrt{2}/2-1 & -\sqrt{2}/2 \\ 0 & \sqrt{2}/2 & \sqrt{2}/2-1 \end{pmatrix} P = \begin{pmatrix} -6 \\ 0 \\ 0 \end{pmatrix} \rightsquigarrow P=A$$

(c) $\sigma: 2x+3y+5z+7=0 \qquad n=(2,3,5)\perp\sigma \qquad P=(-1,0,-1)\in\sigma$

$n'' = (-2,-\sqrt{2},5\sqrt{2}) \qquad P'' = (1,\sqrt{2}/2,-\sqrt{2}/2)+(6,0,0) = (7,\sqrt{2}/2,-\sqrt{2}/2)$

$\sigma'': -2x-\sqrt{2}y+5\sqrt{2}z+d''=0 \quad P''\in\sigma'' \rightsquigarrow -15-1-5+d''=0 \quad d''=19$

$$-2x-\sqrt{2}y+5\sqrt{2}z+19=0$$

$r: (1,2,3)+\delta(-1,2,1) = P_0+\delta V \qquad V'' = (1,\sqrt{2}/2,3\sqrt{2}/2)$

$P_0'' = (-1,-\sqrt{2}/2,3\sqrt{2}/2)+(6,0,0) = (5,-\sqrt{2}/2,5\sqrt{2}/2)$

$r'': (5,-\sqrt{2}/2,5\sqrt{2}/2)+\delta(1,\sqrt{2}/2,3\sqrt{2}/2)$

(d) $P''=SP+2A \qquad P=S^T(P''-2A) \qquad S^T=\begin{pmatrix}-1&0&0\\0&\sqrt2/2&\sqrt2/2\\0&-\sqrt2/2&\sqrt2/2\end{pmatrix}$

$\delta'': 2x+3y+5z+7=0 \qquad n''=(2,3,5) \qquad P''=(-1,0,-1)$

$n=(-2,5\sqrt2,\sqrt2) \qquad P=(7,-\sqrt2/2,-\sqrt2/2)$

$\delta: -2x+5\sqrt2 y+\sqrt2 z+d=0 \quad P\in\delta \rightsquigarrow -15-5-1+d=0 \quad d=19$

$-2x+5\sqrt2 y+\sqrt2 z+19=0$

$r'': (1,2,3)+\delta(-1,2,1) \quad v=(1,3\sqrt2/2,-\sqrt2/2) \quad P=(5,5\sqrt2/2,\sqrt2/2)$

$r: (5,5\sqrt2/2,\sqrt2/2)+\delta(1,3\sqrt2/2,-\sqrt2/2)$

(e) $c: x^2+y^2+z^2+3x-2y=10 \qquad x^2+y^2+z^2-2ax-2by-2cz=R^2-a^2-b^2-c^2$

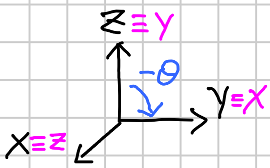
CENTRO $C=\left(\overset{a}{-\tfrac32},\overset{b}{1},\overset{c}{0}\right) \qquad R^2=10+\overset{a^2}{\tfrac94}+\overset{b^2}{1}=\tfrac{53}{4} \qquad R=\tfrac{\sqrt{53}}{2}$

$C''=\left(\tfrac32,\tfrac{\sqrt2}{2},\tfrac{\sqrt2}{2}\right) \qquad c'': x^2+y^2+z^2-3x-\sqrt2 y-\sqrt2 z=\tfrac{53}{4}-\tfrac94-\tfrac24-\tfrac24$

$x^2+y^2+z^2-3x-\sqrt2 y-\sqrt2 z=10$

(15) rotazione, intorno alla retta passante per $(1,2,3)$ e parallela all'asse x, di 30° in un verso giudicato orario da un omino orientato secondo il semiasse positivo delle x.

(a) $P'=S(P-A)+A$



$$S=\overset{M}{\begin{pmatrix}0&0&1\\1&0&0\\0&1&0\end{pmatrix}}\overset{\hat S}{\begin{pmatrix}\sqrt3/2&-1/2&0\\1/2&\sqrt3/2&0\\0&0&1\end{pmatrix}}\overset{M^{-1}}{\begin{pmatrix}0&1&0\\0&0&1\\1&0&0\end{pmatrix}}=$$

$$=\begin{pmatrix}0&0&1\\\sqrt3/2&-1/2&0\\1/2&\sqrt3/2&0\end{pmatrix}\begin{pmatrix}0&1&0\\0&0&1\\1&0&0\end{pmatrix} \rightsquigarrow S=\begin{pmatrix}1&0&0\\0&\sqrt3/2&-1/2\\0&1/2&\sqrt3/2\end{pmatrix} \qquad A=(0,2,3)$$

(b) $P'=S(P-A)+A=P \rightsquigarrow (S-I)(P-A)=\begin{pmatrix}0&0&0\\0&\frac{\sqrt3}{2}-1&-1/2\\0&1/2&\frac{\sqrt3}{2}-1\end{pmatrix}(P-A)=0 \qquad P=(\delta,2,3)$

(c) $\pi: 2x+3y+5z+7=0 \qquad n=(2,3,5)\perp\pi \qquad P=(-1,0,-1)\in\pi$

$n'=(2, 3\sqrt{3}/2-5/2, 3/2+5\sqrt{3}/2)\equiv(4, 3\sqrt{3}-5, 3+5\sqrt{3}) \qquad (P-A)=(-1,-2,-4)$

$P'=(-1,-\sqrt{3}+2,-1-2\sqrt{3})+(0,2,3)=(-1, 4-\sqrt{3}, 2-2\sqrt{3})$

$\pi': 4x+(3\sqrt{3}-5)y+(3+5\sqrt{3})z+d'=0$

$P'\in\pi' \rightsquigarrow -4+12\sqrt{3}-20-9+5\sqrt{3}+6+10\sqrt{3}-6\sqrt{3}-30+d'=0 \qquad d'=57-21\sqrt{3}$

$4x+(3\sqrt{3}-5)y+(3+5\sqrt{3})z+57-21\sqrt{3}=0$

$r: (1,2,3)+\sigma(-1,2,1)=P_0+\sigma v \qquad v'=(-1,\sqrt{3}-1/2, 1+\sqrt{3}/2)$

$(P_0-A)=(1,0,0) \qquad P_0'=(1,0,0)+A=(1,2,3)=P_0$

$r': (1,2,3)+\sigma(-1,\sqrt{3}-1/2,1+\sqrt{3}/2)$

(d) $P'=S(P-A)+A \rightsquigarrow P=S^{\delta}(P'-A)+A \qquad S^{\delta}=\begin{pmatrix}1&0&0\\0&\sqrt{3}/2&1/2\\0&-1/2&\sqrt{3}/2\end{pmatrix}$

$\pi': 2x+3y+5z+7=0 \qquad n'=(2,3,5) \qquad P'=(-1,0,-1)$

$n=(2,3\sqrt{3}/2+5/2,-3/2+5\sqrt{3}/2)\equiv(4,3\sqrt{3}+5,-3+5\sqrt{3})$

$(P'-A)=(-1,-2,-4) \qquad P=(-1,-\sqrt{3}-2,1-2\sqrt{3})+(0,2,3)=(-1,-\sqrt{3},4-2\sqrt{3})$

$\pi: 4x+(3\sqrt{3}+5)y+(-3+5\sqrt{3})z+d=0$

$P\in\pi \rightsquigarrow -4-9-5\sqrt{3}-12+20\sqrt{3}+6\sqrt{3}-30+d=0 \qquad d=55-21\sqrt{3}$

$4x+(3\sqrt{3}+5)y+(-3+5\sqrt{3})z+55-21\sqrt{3}=0$

$r': \overset{P_0'}{(1,2,3)}+\sigma\overset{v'}{(-1,2,1)} \qquad v=(-1,\sqrt{3}+1/2,-1+\sqrt{3}/2) \qquad P=(1,2,3)$

$r: (1,2,3)+\sigma(-1,\sqrt{3}+1/2,-1+\sqrt{3}/2)$

(e) $C: x^2+y^2+z^2+3x-2y=10$ $\qquad x^2+y^2+z^2-2ax-2by-2cz=R^2-a^2-b^2-c^2$

CENTRO $C=(\overset{a}{-\frac{3}{2}}, \overset{b}{1}, \overset{c}{0})$ $\qquad R^2=10+\overset{a^2}{\frac{9}{4}}+\overset{b^2}{1}=\frac{53}{4}$ $\qquad R=\frac{\sqrt{53}}{2}$

$C'=\left(-\frac{3}{2}, \frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ $\qquad c': x^2+y^2+z^2+3x-\sqrt{3}y-z=\frac{53}{4}-\frac{9}{4}-\frac{3}{4}-\frac{1}{4}$

$$x^2+y^2+z^2+3x-\sqrt{3}y-z=10$$