(b) $\langle p(x), q(x)\rangle = p(0)q(0) + 3\,p(1)q(1) + 5\,p(2)q(2)$

$$B = \begin{pmatrix} \langle 1,1\rangle & \langle 1,x\rangle & \langle 1,x^2\rangle \\ \langle x,1\rangle & \langle x,x\rangle & \langle x,x^2\rangle \\ \langle x^2,1\rangle & \langle x^2,x\rangle & \langle x^2,x^2\rangle \end{pmatrix} \qquad \begin{aligned} \langle 1,1\rangle &= 1+3+5 = 9 \\ \langle 1,x\rangle &= 0+3+10=13 \\ \langle 1,x^2\rangle &= 0+3+20=23 \end{aligned}$$

$$\begin{aligned} \langle x,x\rangle &= 0+3+20=23 \\ \langle x,x^2\rangle &= 0+3+40=43 \\ \langle x^2,x^2\rangle &= 0+3+80=83 \end{aligned} \rightsquigarrow B = \begin{pmatrix} 9 & 13 & 23 \\ 13 & 23 & 43 \\ 23 & 43 & 83 \end{pmatrix}$$

NELLA BASE $\{1, x, x^2\}$

(c) BASE ORTOGONALE

$$\begin{cases} w_1 = 1 \qquad \hat{w}_2 = x - \dfrac{\langle x,1\rangle}{\langle 1,1\rangle}\,1 = x - \dfrac{13}{9} \qquad w_2 = 9x-13 \\ w_3 = x^2 - \dfrac{\langle x^2,1\rangle}{\langle 1,1\rangle}\,1 - \dfrac{\langle x^2,w_2\rangle}{\langle w_2,w_2\rangle}\,w_2 \end{cases}$$

$$\begin{cases} \langle x^2,w_2\rangle = (x^2)^T B\, w_2 = (23 \;\; 43 \;\; 83)\begin{pmatrix} -13 \\ 9 \\ 0 \end{pmatrix} = -299+387 = 88 \\ \langle w_2,w_2\rangle = w_2^T B\, w_2 = (0 \;\; -169+207 \;\; -299+387)\begin{pmatrix} -13 \\ 9 \\ 0 \end{pmatrix} = 342 \end{cases}$$

$$\rightsquigarrow \hat{w}_3 = x^2 - \frac{23}{9} - \frac{\overset{44}{\cancel{88}}}{\underset{171}{\cancel{342}}}(9x-13) = x^2 - \frac{23}{9} - \frac{\overset{\overset{44}{\cancel{132}}}{\cancel{396}}}{\underset{\underset{19}{\cancel{57}}}{\cancel{171}}}x + \frac{572}{171} = x^2 - \frac{44}{19}x + \frac{572-437}{171}$$

$$= x^2 - \frac{44}{19}x + \frac{135}{171} \qquad w_3 = 171x^2 - 396x + 135$$

$$\langle w_3,w_3\rangle = w_3^T B\, w_3 = (9\cdot135-13\cdot396+23\cdot171 \;\; 13\cdot135+23\cdot396+43\cdot171 \;\; 23\cdot135+43\cdot396+83\cdot171)\begin{pmatrix} 135 \\ -396 \\ 171 \end{pmatrix}$$

$$= (0 \;\; 0 \;\; 270)\begin{pmatrix} 135 \\ -396 \\ 171 \end{pmatrix} = 46170$$

$$
\begin{cases}
w_1^* = \dfrac{w_1}{\|w_1\|} = \dfrac{1}{3} \\[2ex]
w_2^* = \dfrac{w_2}{\|w_2\|} = \dfrac{1}{\sqrt{352}}\left(-13+9x\right) \\[2ex]
w_3^* = \dfrac{w_3}{\|w_3\|} = \dfrac{1}{\sqrt{56170}}\left(135-336x+171x^2\right)
\end{cases}
$$

$$
M = \begin{array}{c}
\begin{array}{ccc} \quad w_1^* \quad & \quad w_2^* \quad & \quad w_3^* \quad \end{array} \\
\begin{pmatrix}
1/3 & -13/\sqrt{352} & 135/\sqrt{56170} \\
0 & 9/\sqrt{352} & -336/\sqrt{56170} \\
0 & 0 & 171/\sqrt{56170}
\end{pmatrix}
\end{array}
\rightsquigarrow \quad M^{\delta} B M = I
$$