$$\lim_{x\to 0} \frac{\log(\cos x)}{x^2} = \lim_{x\to 0} \frac{\log(1+(\cos x-1))}{x^2}$$

$$= \lim_{x\to 0} \frac{\log(1+(\cos x-1))}{x^2} \cdot \frac{\cos x-1}{\cos x-1} = \lim_{x\to 0} \frac{\cos x-1}{x^2} = -\frac{1}{2}$$

$$\downarrow$$

$$\lim \frac{\log(1+(\cos x-1))}{\cos x-1} \to 1$$