Forme quadratiche 1

Argomenti: segnatura di forme quadratiche **Difficoltà**: ⋆⋆

Prerequisiti: criteri per la segnatura (completamento quadrati, Sylvester, Cartesio)

Determinare la segnatura delle seguenti forme quadratiche, cioè la terna (n_0, n_+, n_-) che rappresenta, sostanzialmente, il numero di autovalori nulli, positivi, negativi. Dedurne se la forma è (semi)definita positiva/negativa o indefinita. Determinare anche un sottospazio di dimensione n_+ su cui la forma è definita positiva, ed un sottospazio di dimensione n_- su cui è definita negativa. Per meglio familiarizzare con le tecniche, si consiglia di determinare la segnatura utilizzando, ove possibile e almeno per le prime volte, almeno 3 metodi (completamento dei quadrati, Sylvester in varie direzioni, Cartesio).

1. Forme quadratiche $q(x, y)$ in $\mathbb{R}^2$:

$x^2 + y^2$	$3x^2 + y^2$	$x^2 - y^2$	$2x^2 - 3y^2$
$x^2 + y^2 + xy$	$x^2 + y^2 + 2xy$	$x^2 + y^2 + 3xy$	$x^2 + 2y^2 + 3xy$
$x^2 + y^2 - xy$	$x^2 + y^2 - 2xy$	$x^2 + y^2 - 3xy$	$x^2 + 2y^2 - 3xy$
$x^2 - y^2 + xy$	$x^2 - y^2 + 2xy$	$x^2 - y^2 + 3xy$	$x^2 - 2y^2 + 3xy$
$7xy$	$-x^2 - 2y^2 + 10xy$	$-y^2 - 7xy$	$3x^2 + 16y^2 - 14xy$
$-y^2$	$x^2 + xy$	$5x^2 + 7y^2 + 12xy$	$-3x^2 - 16y^2 - 14xy$

2. Forme quadratiche $q(x, y, z)$ in $\mathbb{R}^3$:

$x^2 + 3y^2 + 5z^2$	$-x^2 + 3y^2 + 5z^2$	$x^2 - 3y^2 - 5z^2$
$x^2 + y^2 + yz$	$x^2 + y^2 - 2yz$	$x^2 + y^2 + 3yz$
$x^2 - y^2 - 3yz$	$z^2 + 3yz + 5xz$	$2xy - y^2 - z^2$

$x^2 + 2y^2 + 3z^2 + 4xy + 5xz + 6yz$	$x^2 - 2y^2 + 3z^2 + 4xy + 5xz + 6yz$
$x^2 + y^2 + z^2 + 2xy + 2yz + 2xz$	$x^2 - y^2 + z^2 + 2xy + 2yz + 2xz$
$x^2 + y^2 + z^2 + 2xy + 2yz - 2xz$	$x^2 + y^2 + z^2 + 2xy + 2yz + 3xz$
$x^2 + y^2 + z^2 + xy + 2yz + 3xz$	$xy - 2yz +$ ~~$3yz$~~ $3xz$
$-y^2 + z^2 + xy - yz$	$y^2 + 7xz$

3. Forme quadratiche $q(x, y, z, w)$ in $\mathbb{R}^4$:

$x^2 - w^2$	$x^2 - 2y^2 + 3z^2 - 4w^2$	$y^2 - 3z^2 - 5w^2$
$x^2 + yz - w^2$	$x^2 + y^2 + w^2 + zx$	$-x^2 + z^2 + w^2 - zy$
$-y^2 - w^2 - 3xz$	$2y^2 + 3z^2 - 5w^2 + xz - 3yw$	$xy - 3zw$

$x^2 + y^2 - z^2 - w^2 - 2xy - 2zw$	$x^2 + y^2 - z^2 - w^2 - 2xz - 2yw$

SOLUZIONI?
BY GIMUSI 12/01/15

1. Forme quadratiche $q(x,y)$ in $\mathbb{R}^2$:

(a) x^2+y^2	(b) $3x^2+y^2$	(c) x^2-y^2	(α) $2x^2-3y^2$
(e) x^2+y^2+xy	(f) x^2+y^2+2xy	(g) x^2+y^2+3xy	(h) x^2+2y^2+3xy
(i) x^2+y^2-xy	(l) x^2+y^2-2xy	(m) x^2+y^2-3xy	(n) x^2+2y^2-3xy
(o) x^2-y^2+xy	(p) x^2-y^2+2xy	(q) x^2-y^2+3xy	(r) x^2-2y^2+3xy
(s) $7xy$	(t) $-x^2-2y^2+10xy$	(u) $-y^2-7xy$	(v) $3x^2+16y^2-14xy$
(w) $-y^2$	(x) x^2+xy	(y) $5x^2+7y^2+12xy$	(z) $-3x^2-16y^2-14xy$

(a) $x^2+y^2 \rightsquigarrow \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ $\lambda_1=\lambda_2=1 \rightsquigarrow (n_0,n_+,n_-)=(0,2,0) \rightsquigarrow$ DEF. POSITIVA

COMPLET. DEI QUADRATI: $x^2+y^2>0 \quad x,y\neq 0$ SYLVESTER: $+\overset{P}{+}\overset{P}{+} \rightsquigarrow n_+=2$

CARTESIO: $(1-\lambda^2)=\lambda^2\overset{V}{-}2\overset{V}{\lambda}+1 \rightsquigarrow n_0=0,\ n_+=2,\ n_-=2-2=0$

(b) $3x^2+y^2 \rightsquigarrow \begin{pmatrix} 3 & 0 \\ 0 & 1 \end{pmatrix}$ $\begin{cases}\lambda_1=3\\ \lambda_2=1\end{cases} \rightsquigarrow (n_0,n_+,n_-)=(0,2,0) \rightsquigarrow$ DEF. POSITIVA

CQ: $(\sqrt{3}x)^2+y^2>0 \quad x,y\neq 0$ SY: $+\overset{P}{+}\overset{P}{+} \rightsquigarrow n_+=2$ CAR: $(1-\lambda)(3-\lambda)=\lambda^2\overset{V}{-}4\overset{V}{\lambda}+3 \rightsquigarrow n_0=0\ n_+=2$

(c) $x^2-y^2 \rightsquigarrow \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ $\begin{cases}\lambda_1=1\\ \lambda_2=-1\end{cases} \rightsquigarrow (n_0,n_+,n_-)=(0,1,1) \rightsquigarrow$ NON DEFINITA

CQ: $x^2-y^2>0\ v=\begin{pmatrix}1\\0\end{pmatrix} \quad x^2-y^2<0\ v=\begin{pmatrix}0\\1\end{pmatrix}$ SY: $(1,2) \rightsquigarrow +\overset{P}{+}\overset{V}{-} \quad n_+=1\ n_-=1 \quad (2,1)\rightsquigarrow +\overset{V}{-}\overset{P}{-}$

CAR: $(1-\lambda)(-1-\lambda)=\lambda^2\overset{V}{-}1 \rightsquigarrow n_0=0,\ n_+=1,\ n_-=2-1=1$

(α) $2x^2-3y^2 \rightsquigarrow \begin{pmatrix} 2 & 0 \\ 0 & -3 \end{pmatrix}$ $\begin{cases}\lambda_1=2\\ \lambda_2=-3\end{cases} \rightsquigarrow (n_0,n_+,n_-)=(0,1,1) \rightsquigarrow$ NON DEFINITA

CQ: $2x^2-3y^2>0\ v=\begin{pmatrix}1\\0\end{pmatrix} \quad x^2-3y^2<0\ v=\begin{pmatrix}0\\1\end{pmatrix}$ SY: $(1,2)\rightsquigarrow +\overset{P}{+}\overset{V}{-} \quad n_+=1\ n_-=1 \quad (2,1)\rightsquigarrow +\overset{V}{-}\overset{P}{-}$

CAR: $(2-\lambda)(-3-\lambda)=\lambda^2\overset{P}{+}\overset{V}{\lambda}-6 \rightsquigarrow n_0=0,\ n_+=1,\ n_-=2-1=1$

(e) $x^2+y^2+xy \rightsquigarrow \begin{pmatrix} 1 & 1/2 \\ 1/2 & 1 \end{pmatrix}$ $|A-\lambda I|=\begin{vmatrix} 1-\lambda & 1/2 \\ 1/2 & 1-\lambda \end{vmatrix}=\begin{cases}(1-\lambda)^2-1/4=\lambda^2-2\lambda+3/4=0\\ \lambda=1\pm 1/2 \quad \lambda_1=3/2\ \ \lambda_2=1/2\end{cases}$ $\begin{matrix} n_0 & n_+ & n_- \\ (0, & 2, & 0)\end{matrix}$ DEF. POS.

CQ: $x^2+xy+\frac{y^2}{4}-\frac{y^2}{4}+y^2=\left(x+\frac{y}{2}\right)^2+\frac{3}{4}y^2>0 \quad \forall v\neq 0$

SY: $(1,2)\rightsquigarrow +\overset{P}{+}\overset{P}{+} \quad n_+=2$ CAR: $\lambda^2\overset{V}{-}2\overset{V}{\lambda}+3/4 \quad n_0=0\ n_+=2\ n_-=2-2=0$

(f) $x^2+y^2+2xy \rightsquigarrow \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$ $|A-\lambda I|=\begin{vmatrix} 1-\lambda & 1 \\ 1 & 1-\lambda \end{vmatrix}=\begin{cases}(1-\lambda)^2-1=\lambda^2-2\lambda=0\\ \lambda_1=0\ \ \lambda_2=2\end{cases}$ $\begin{matrix} n_0 & n_+ & n_- \\ (1, & 1, & 0)\end{matrix}$ SEMIDEF. POS.

CQ: $(x+y)^2\geq 0 \quad (x+y)^2=0\ x=-y$ SY: NON APPLICABILE ($\det A=0$)

CAR: $\lambda^2\overset{V}{-}2\lambda \quad n_0=1\ n_+=1\ n_-=2-1-1=0$

(g) $x^2+y^2+3xy \rightsquigarrow \begin{pmatrix} 1 & 3/2 \\ 3/2 & 1 \end{pmatrix}$ $|A-\lambda I| = \begin{vmatrix} 1-\lambda & 3/2 \\ 3/2 & 1-\lambda \end{vmatrix} = \begin{cases} (1-\lambda)^2 - 9/4 = \lambda^2-2\lambda-5/4=0 \\ \lambda = \frac{2\pm\sqrt{4+5}}{2} \end{cases}$ $\begin{cases} \lambda_1 = 5/2 \\ \lambda_2 = -1/2 \end{cases}$ $m_0\ m_+\ m_-$ $(0,1,1)$ NON DEF.

CQ: $(x+y)^2+xy = (x+y)^2 + \frac{1}{4}(x+y)^2 - \frac{1}{4}(x-y)^2 = \frac{5}{4}(x+y)^2 - \frac{1}{4}(x-y)^2$

$q(v)\geq 0$ $x=y=\delta \equiv (\delta,\delta)$, $q(v)\leq 0$ $x=-y=\delta \equiv (\delta,-\delta)$

SY: $(1,2) \rightsquigarrow + \overset{P}{+} \overset{V}{-}$ $m_+=1$ $m_-=1$ CAR: $\lambda^{\overset{V}{2}} \overset{P}{-} 2\lambda - 5/4$ $m_0=0$ $m_+=1$ $m_-=2-1=1$

(h) $x^2+2y^2+3xy \rightsquigarrow \begin{pmatrix} 1 & 3/2 \\ 3/2 & 2 \end{pmatrix}$ $|A-\lambda I| = \begin{vmatrix} 1-\lambda & 3/2 \\ 3/2 & 2-\lambda \end{vmatrix} = \begin{cases} (1-\lambda)(2-\lambda)-9/4 = \lambda^2-3\lambda-1/4=0 \\ \lambda = \frac{3\pm\sqrt{10}}{2} \end{cases}$ $\begin{cases} \lambda_1 = (3+\sqrt{10})/2 > 0 \\ \lambda_2 = (3-\sqrt{10})/2 < 0 \end{cases}$ N.D.

CQ: $(x+\frac{3}{2}y)^2 - \frac{9}{4}y^2+2y^2 = (x+\frac{3}{2}y)^2 - \frac{1}{4}y^2 \rightsquigarrow$ $q(v)\geq 0$ $(\delta,0)$, $q(v)\leq 0$ $(\delta,-\frac{2}{3}\delta)$

SY: $(1,2)$ $+ \overset{P}{+} \overset{V}{-}$ $m_+=1$ $m_-=1$ CAR: $\lambda^{\overset{V}{2}} \overset{P}{-} 3\lambda - 1/4$ $m_0=0$ $m_+=1$ $m_-=2-1-0=1$

(i) $x^2+y^2-xy \rightsquigarrow \begin{pmatrix} 1 & -1/2 \\ -1/2 & 1 \end{pmatrix}$ $|A-\lambda I| = \begin{vmatrix} 1-\lambda & -1/2 \\ -1/2 & 1-\lambda \end{vmatrix} = \begin{cases} (1-\lambda)^2 - 1/4 = \lambda^2-2\lambda+3/4=0 \\ \lambda = \frac{2\pm\sqrt{1}}{2} \end{cases}$ $\lambda_1=3/2$ $\lambda_2=1/2$ $m_0\ m_+\ m_-$ $(0,2,0)$ DEF. POS.

CQ: $(x-\frac{y}{2})^2 - \frac{y^2}{4} + y^2 = (x-\frac{y}{2})^2 + \frac{3}{4}y^2 \rightsquigarrow$ $q(v)>0$ $v_1=(\delta,0)$ $v_2=(\delta,2\delta)$ $\delta\neq 0$

SY: $(1,2)$ $+ \overset{P}{+} \overset{P}{+}$ $m_+=2$ CAR: $\lambda^{\overset{V}{2}} \overset{V}{-} 2\lambda + 3/4$ $m_0=0$ $m_+=2$ $m_0=2-2=0$

(l) $x^2+y^2-2xy \rightsquigarrow \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$ $|A-\lambda I| = \begin{vmatrix} 1-\lambda & -1 \\ -1 & 1-\lambda \end{vmatrix} = \begin{cases} (1-\lambda)^2 - 1 = \lambda^2-2\lambda=0 \\ \lambda_1=0\ \ \lambda_2=2 \end{cases}$ $m_0\ m_+\ m_-$ $(1,1,0)$ SEMIDEF. POS.

CQ: $(x-y)^2 \rightsquigarrow$ $q(v)>0$ $v=(\delta,0)$ $\delta\neq 0$ SY: NON APPLICABILE (DET=0)

CAR: $\lambda^{\overset{V}{2}} - 2\lambda$ $m_0=1$ $m_+=1$ $m_-=2-1-1=0$

(m) $x^2+y^2-3xy \rightsquigarrow \begin{pmatrix} 1 & -3/2 \\ -3/2 & 1 \end{pmatrix}$ $|A-\lambda I| = \begin{vmatrix} 1-\lambda & -3/2 \\ -3/2 & 1-\lambda \end{vmatrix} = \begin{cases} (1-\lambda)^2 - 9/4 = \lambda^2-2\lambda-5/4=0 \\ \lambda = \frac{2\pm\sqrt{9}}{2} \end{cases}$ $\lambda_1=5/2$ $\lambda_2=-1/2$ $m_0\ m_+\ m_-$ $(0,1,1)$ NON DEF.

CQ: $(x-\frac{3}{2}y)^2 - \frac{9}{4}y^2+y^2 = (x-\frac{3}{2}y)^2 - \frac{5}{4}y^2 \rightsquigarrow$ $q(v)>0$ $v=(\delta,0)$, $q(v)<0$ $v=(\frac{3}{2}\delta,\delta)$ $\delta\neq 0$

SY: $(1,2)$ $+ \overset{P}{+} \overset{V}{-}$ $m_+=1$ $m_-=1$ CAR: $\lambda^{\overset{V}{2}} \overset{P}{-} 2\lambda - 5/4$ $m_0=0$ $m_+=1$ $m_-=2-1=1$

(n) $x^2+2y^2-3xy \rightsquigarrow \begin{pmatrix} 1 & -3/2 \\ -3/2 & 2 \end{pmatrix}$ $|A-\lambda I| = \begin{vmatrix} 1-\lambda & -3/2 \\ -3/2 & 2-\lambda \end{vmatrix} = \begin{cases} (1-\lambda)(2-\lambda)-9/4 = \lambda^2-3\lambda-1/4=0 \\ \lambda = \frac{3\pm\sqrt{10}}{2} \end{cases}$ $\lambda_{1,2} = \frac{3}{2}\pm\frac{\sqrt{10}}{2}$ $m_0\ m_+\ m_-$ $(0,1,1)$ NON DEF.

CQ: $(x-\frac{3}{2}y)^2 - \frac{9}{4}y^2+2y^2 = (x-\frac{3}{2}y)^2 - \frac{1}{4}y^2 \rightsquigarrow$ $q(v)>0$ $v=(\delta,0)$, $q(v)<0$ $v=(\frac{3}{2}\delta,\delta)$ $\delta\neq 0$

SY: $(1,2)$ $+ \overset{P}{+} \overset{V}{-}$ $m_+=1$ $m_-=1$ CAR: $\lambda^{\overset{V}{2}} \overset{P}{-} 3\lambda - 1/4$ $m_0=0$ $m_+=1$ $m_-=2-1=1$

(σ) $x^2-y^2+xy \rightsquigarrow \begin{pmatrix} 1 & 1/2 \\ 1/2 & -1 \end{pmatrix} \quad |A-\lambda I| = \begin{vmatrix} 1-\lambda & 1/2 \\ 1/2 & -1-\lambda \end{vmatrix} = \begin{cases} (1-\lambda)(-1-\lambda)-1/4 = \lambda^2 - 5/4 = 0 \\ \lambda_{1,2} = \pm \frac{1}{2}\sqrt{5} \end{cases}$ $m_0\ m_+\ m_-$ (0, 1, 1) NON DEF.

CQ: $\left(x+\frac{y}{2}\right)^2 - \frac{y^2}{4} - y^2 = \left(x+\frac{y}{2}\right)^2 - \frac{5}{4}y^2 \rightsquigarrow q(v)>0\ v=(\delta,0),\ q(v)<0\ v=\left(-\frac{\delta}{2},\delta\right)\ \delta\neq 0$

SY: $(1,2)\ +\ \overset{P}{+}\ \overset{V}{-} \quad (2,2)\ +\ \overset{V}{-}\ \overset{P}{-} \quad m_+=1\ m_-=1$ CAR: $\lambda^{\overset{V}{2}} - 5/4 \quad m_0=0\ m_+=1\ m_-=2-1=1$

(ρ) $x^2-y^2+2xy \rightsquigarrow \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \quad |A-\lambda I| = \begin{vmatrix} 1-\lambda & 1 \\ 1 & -1-\lambda \end{vmatrix} = \begin{cases} \lambda^2 - 1 - 1 = \lambda^2 - 2 = 0 \\ \lambda_{1,2} = \pm\sqrt{2} \end{cases}$ $m_0\ m_+\ m_-$ (0, 1, 1) NON DEF.

CQ: $(x+y)^2 - 2y^2 \rightsquigarrow q(v)>0\ v=(\delta,0),\ q(v)<0\ v=(\delta,-\delta)\ \delta\neq 0$

SY: $(1,2)\ +\ \overset{P}{+}\ \overset{V}{-} \quad (2,2)\ +\ \overset{V}{-}\ \overset{P}{-} \quad m_+=1\ m_-=1$ CAR: $\lambda^{\overset{V}{2}} - 2 \quad m_0=0\ m_+=1\ m_-=2-1=1$

(q) $x^2-y^2+3xy \rightsquigarrow \begin{pmatrix} 1 & 3/2 \\ 3/2 & -1 \end{pmatrix} \quad |A-\lambda I| = \begin{vmatrix} 1-\lambda & 3/2 \\ 3/2 & -1-\lambda \end{vmatrix} = \begin{cases} \lambda^2 - 1 - 9/4 = \lambda^2 - 13/4 = 0 \\ \lambda_{1,2} = \pm\sqrt{13}/2 \end{cases}$ $m_0\ m_+\ m_-$ (0, 1, 1) NON DEF.

CQ: $\left(x+\frac{3}{2}y\right)^2 - \frac{9}{4}y^2 - y^2 = \left(x+\frac{3}{2}y\right)^2 - \frac{13}{4}y^2 \rightsquigarrow q(v)>0\ v=(\delta,0),\ q(v)<0\ v=\left(-\frac{3}{2}\delta,\delta\right)\ \delta\neq 0$

SY: $(1,2)\ +\ \overset{P}{+}\ \overset{V}{-} \quad (2,2)\ +\ \overset{V}{-}\ \overset{P}{-} \quad m_+=1\ m_-=1$ CAR: $\lambda^{\overset{V}{2}} - 13/4 \quad m_0=0\ m_+=1\ m_-=2-1=1$

(r) $x^2-2y^2+3xy \rightsquigarrow \begin{pmatrix} 1 & 3/2 \\ 3/2 & -2 \end{pmatrix} \quad |A-\lambda I| = \begin{vmatrix} 1-\lambda & 3/2 \\ 3/2 & -2-\lambda \end{vmatrix} = \begin{cases} (1-\lambda)(-2-\lambda) - 9/4 = \lambda^2+\lambda-\frac{17}{4} = 0 \\ \lambda = \frac{-1\pm\sqrt{18}}{2} \quad \lambda_{1,2} = -\frac{1}{2} \pm \frac{3}{2}\sqrt{2} \end{cases}$ $m_0\ m_+\ m_-$ (0, 1, 1) NON DEF.

CQ: $\left(x+\frac{3}{2}y\right)^2 - \frac{9}{4}y^2 - 2y^2 = \left(x+\frac{3}{2}y\right)^2 - \frac{17}{4}y^2 \rightsquigarrow q(v)>0\ v=(\delta,0),\ q(v)<0\ v=\left(-\frac{3}{2}\delta,\delta\right)\ \delta\neq 0$

SY: $(1,2)\ +\ \overset{P}{+}\ \overset{V}{-} \quad (2,2)\ +\ \overset{V}{-}\ \overset{P}{-} \quad m_+=1\ m_-=1$ CAR: $\lambda^{\overset{V}{2}} \overset{P}{-} \lambda - \frac{17}{4} \quad m_0=0\ m_+=1\ m_-=2-1=1$

(s) $7xy \rightsquigarrow \begin{pmatrix} 0 & 7/2 \\ 7/2 & 0 \end{pmatrix} \quad |A-\lambda I| = \begin{vmatrix} -\lambda & 7/2 \\ 7/2 & -\lambda \end{vmatrix} = \begin{cases} \lambda^2 - 49/4 = 0 \\ \lambda_{1,2} = \pm 7/2 \end{cases}$ $m_0\ m_+\ m_-$ (0, 1, 1) NON DEF.

CQ: $\frac{7}{4}(x+y)^2 - \frac{7}{4}(x-y)^2 \rightsquigarrow q(v)>0\ v=(\delta,\delta),\ q(v)<0\ v=(\delta,-\delta)\ \delta\neq 0$

SY: NON APPLICABILE CAR: $\lambda^{\overset{V}{2}} - 49/4 \quad m_0=0\ m_+=1\ m_-=2-1=1$

(δ) $-x^2-2y^2+10xy \rightsquigarrow \begin{pmatrix} -1 & 5 \\ 5 & -2 \end{pmatrix} \quad |A-\lambda I| = \begin{vmatrix} -1-\lambda & 5 \\ 5 & -2-\lambda \end{vmatrix} = \begin{cases} (-1-\lambda)(-2-\lambda) - 25 = \lambda^2+3\lambda-23 = 0 \\ \lambda_{1,2} = \frac{-3\pm\sqrt{101}}{2} \end{cases}$ $m_0\ m_+\ m_-$ (0, 1, 1) NON DEF.

CQ: $-(x-5y)^2 + 23y^2 \rightsquigarrow q(v)>0\ v=(5\delta,\delta)\quad q(v)<0\ v=(\delta,0)\ \delta\neq 0$

SY: $(1,2)\ +\ \overset{V}{-}\ \overset{P}{-} \quad m_+=1\ m_-=1$ CAR: $\lambda^2 \overset{P}{+} 3\lambda \overset{V}{-} 23 \quad m_0=0\ m_+=1\ m_-=2-1=1$

(μ) $-y^2-7xy \rightsquigarrow \begin{pmatrix} 0 & -7/2 \\ -7/2 & -1 \end{pmatrix} \quad |A-\lambda I| = \begin{vmatrix} -\lambda & -7/2 \\ -7/2 & -1-\lambda \end{vmatrix} = \begin{cases} \lambda^2+\lambda-49/4 = 0 \\ \lambda_{1,2} = \frac{-1\pm\sqrt{50}}{2} = \frac{-1\pm 5\sqrt{2}}{2} \end{cases}$ $m_0\ m_+\ m_-$ (0, 1, 1) NON DEF.

CQ: $-\left(y+\frac{7}{2}x\right)^2 + \frac{49}{4}x^2 \rightsquigarrow q(v)>0\ v=\left(-\frac{7}{2}\delta,\delta\right)\quad q(v)<0\ v=(0,\delta)\ \delta\neq 0$

SY: $(2,2)\ \overset{V}{+}\ \overset{P}{-}\ - \quad m_+=1\ m_-=1$ CAR: $\lambda^2 \overset{P}{+} \lambda \overset{V}{-} 49/4 \quad m_0=0\ m_+=1\ m_-=2-1=1$

(v) $3x^2+16y^2-15xy \rightsquigarrow \begin{pmatrix} 3 & -7 \\ -7 & 16 \end{pmatrix}$ $|A-\lambda I| = \begin{vmatrix} 3-\lambda & -7 \\ -7 & 16-\lambda \end{vmatrix} = \begin{cases} \lambda^2-19\lambda-1=0 \\ \lambda_{1/2}=\frac{19\pm\sqrt{365}}{2} \end{cases}$

$m_0\ m_+\ m_-$
$(0,1,1)$
NON DEF.

CQ: $3\left(x-\frac{7}{3}y\right)^2-\frac{49}{3}y^2+16y^2=3\left(x-\frac{7}{3}y\right)^2-\frac{1}{3}y^2$ $q(v)>0$ $v=(\delta,0)$ $q(v)<0$ $v=\left(\frac{7}{3}\delta,\delta\right)$ $\delta\neq 0$

SY: $+^P+^V-$ $m_+=1$ $m_-=1$ CAR: $\lambda^2\overset{V}{-}19\lambda\overset{P}{-}1$ $m_0=0$ $m_+=1$ $m_-=2-1=1$

(w) $-y^2 \rightsquigarrow \begin{pmatrix} 0 & 0 \\ 0 & -1 \end{pmatrix}$ $|A-\lambda I| = \begin{vmatrix} -\lambda & 0 \\ 0 & -1-\lambda \end{vmatrix} = \begin{cases} \lambda^2+\lambda=0 \\ \lambda_2=0\ \lambda_2=-1 \end{cases}$

$m_0\ m_+\ m_-$
$(1,0,1)$
SEMIDEF. NEG.

CQ: $-y^2 \rightsquigarrow q(v)<0$ $v=(0,\delta)$ $\delta\neq 0$ $q(v)=0$ $v=(\delta,0)$

SY: NON APPLICABILE CAR: $\lambda^2\overset{P}{+}\lambda$ $m_0=1$ $m_+=0$ $m_-=2-1=1$

(x) $x^2+xy \rightsquigarrow \begin{pmatrix} 1 & 1/2 \\ 1/2 & 0 \end{pmatrix}$ $|A-\lambda I| = \begin{vmatrix} 1-\lambda & 1/2 \\ 1/2 & -\lambda \end{vmatrix} = \begin{cases} \lambda^2-\lambda-1/4=0 \\ \lambda_{1,2}=\frac{1\pm\sqrt{2}}{2} \end{cases}$

$m_0\ m_+\ m_-$
$(0,1,1)$
NON DEF.

CQ: $\left(x+\frac{y}{2}\right)^2-\frac{y^2}{4} \rightsquigarrow q(v)>0$ $v=(\delta,0)$ $q(v)<0$ $v=\left(-\frac{\delta}{2},\delta\right)$ $\delta\neq 0$

SY: $(1,2)$ $+^P+^V-$ $m_+=1$ $m_-=1$ CAR: $\lambda^2\overset{V}{-}\lambda\overset{P}{-}1/4$ $m_0=0$ $m_1=1$ $m_2=2-1=1$

(y) $5x^2+7y^2+12xy \rightsquigarrow \begin{pmatrix} 5 & 6 \\ 6 & 7 \end{pmatrix}$ $|A-\lambda I| = \begin{vmatrix} 5-\lambda & 6 \\ 6 & 7-\lambda \end{vmatrix} = \begin{cases} \lambda^2-12\lambda-1=0 \\ \lambda_{1/2}=\frac{12\pm\sqrt{148}}{2}=6\pm\sqrt{37} \end{cases}$

$m_0\ m_+\ m_-$
$(0,1,1)$
NON DEF.

CQ: $5\left(x+\frac{6}{5}y\right)^2-\frac{1}{5}y^2 \rightsquigarrow q(v)>0$ $v=(\delta,0)$ $q(v)<0$ $v=\left(-\frac{6}{5}\delta,\delta\right)$ $\delta\neq 0$

SY: $+^P+^V-$ $m_+=1$ $m_-=1$ CAR: $\lambda^2\overset{V}{-}12\lambda\overset{P}{-}1$ $m_0=0$ $m_+=1$ $m_-=2-1=1$

(z) $-3x^2-16y^2-15xy \rightsquigarrow \begin{pmatrix} -3 & -7 \\ -7 & -16 \end{pmatrix}$ $|A-\lambda I| = \begin{vmatrix} -3-\lambda & -7 \\ -7 & -16-\lambda \end{vmatrix} = \begin{cases} \lambda^2+19\lambda-1=0 \\ \lambda_{1,2}=\frac{-19\pm\sqrt{365}}{2} \end{cases}$

$m_0\ m_+\ m_-$
$(0,1,1)$
NON DEF.

CQ: $-3\left(x+\frac{7}{3}y\right)^2+\frac{49}{3}y^2-16y^2=-3\left(x+\frac{7}{3}y\right)^2+\frac{y^2}{3} \rightsquigarrow q(v)>0$ $v=\left(-\frac{7}{3}\delta,\delta\right)$ $q(v)<0$ $v=(\delta,0)$ $\delta\neq 0$

SY: $+^V-^P-$ $m_+=1$ $m_-=1$ CAR: $\lambda^2\overset{P}{+}19\lambda\overset{V}{-}1$ $m_0=0$ $m_+=1$ $m_-=2-1=1$

2. Forme quadratiche $q(x,y,z)$ in $\mathbb{R}^3$:

(a) $x^2+3y^2+5z^2$ (b) $-x^2+3y^2+5z^2$ (c) $x^2-3y^2-5z^2$

(d) x^2+y^2+yz (e) x^2+y^2-2yz (f) x^2+y^2+3yz

(g) x^2-y^2-3yz (h) $z^2+3yz+5xz$ (i) $2xy-y^2-z^2$

(l) $x^2+2y^2+3z^2+4xy+5xz+6yz$ (m) $x^2-2y^2+3z^2+4xy+5xz+6yz$

(n) $x^2+y^2+z^2+2xy+2yz+2xz$ (o) $x^2-y^2+z^2+2xy+2yz+2xz$

(p) $x^2+y^2+z^2+2xy+2yz-2xz$ (q) $x^2+y^2+z^2+2xy+2yz+3xz$

(r) $x^2+y^2+z^2+xy+2yz+3xz$ (s) $xy-2yz+$ ~~$3yz$~~ $3xz$

(t) $-y^2+z^2+xy-yz$ (u) y^2+7xz

(a) $x^2+3y^2+5z^2 \rightsquigarrow \begin{pmatrix}1&0&0\\0&3&0\\0&0&5\end{pmatrix}$ $\begin{cases}\lambda_1=1\\\lambda_2=3\\\lambda_3=5\end{cases}$ $(n_0,n_+,n_-)=(0,3,0)$ DEF. POSITIVA $q(v)>0\ \forall v\neq 0$

(b) $-x^2+3y^2+5z^2 \rightsquigarrow \begin{pmatrix}-1&0&0\\0&3&0\\0&0&5\end{pmatrix}$ $\begin{cases}\lambda_1=-1\\\lambda_2=3\\\lambda_3=5\end{cases}$ $(n_0,n_+,n_-)=(0,2,1)$ NON DEF. $\begin{cases}q(v)\geq 0 & v=(0,5,5)\\ q(v)\leq 0 & v=(5,0,0)\end{cases}$

(c) $x^2-3y^2-5z^2 \rightsquigarrow \begin{pmatrix}1&0&0\\0&-3&0\\0&0&-5\end{pmatrix}$ $\begin{cases}\lambda_1=1\\\lambda_2=-3\\\lambda_3=-5\end{cases}$ $(0,1,2)$ NON DEF. $\begin{cases}q(v)\geq 0 & v=(5,0,0)\\ q(v)\leq 0 & v=(0,5,5)\end{cases}$

(d) $x^2+y^2+yz \rightsquigarrow \begin{pmatrix}1&0&0\\0&1&1/2\\0&1/2&0\end{pmatrix}$ $|A-\lambda I|=\begin{vmatrix}1-\lambda&0&0\\0&1-\lambda&1/2\\0&1/2&-\lambda\end{vmatrix}=\begin{cases}-\lambda(1-\lambda)^2-\frac{1}{5}(1-\lambda)=\\=(1-\lambda)(-\lambda+\lambda^2-1/5)=\\\lambda_1=1\quad\lambda_{2,3}=\frac{1\pm\sqrt{2}}{2}\end{cases}$

$(n_0,n_+,n_-)=(0,2,1)\rightsquigarrow$ NON DEF.

CQ: $x^2+(y+\frac{z}{2})^2-\frac{z^2}{5}\rightsquigarrow q(v)\geq 0\ v=(5,5,0)\quad q(v)\leq 0\ v=(0,0,5)$

SY: $(1,2,3)$ $+\overset{P}{+}\overset{P}{+}\overset{V}{-}$ $n_+=2$ $n_-=1$ CAR: $-\lambda^3+2\lambda^2-\frac{3}{5}\lambda-\frac{1}{5}$ $n_0=0$ $n_+=2$ $n_-=3-2=1$

(e) $x^2+y^2-2yz \rightsquigarrow \begin{pmatrix}1&0&0\\0&1&-1\\0&-1&0\end{pmatrix}$ $|A-\lambda I|=\begin{vmatrix}1-\lambda&0&0\\0&1-\lambda&-1\\0&-1&-\lambda\end{vmatrix}=\begin{cases}-\lambda(1-\lambda)^2-(1-\lambda)=\\=(1-\lambda)(\lambda^2-\lambda-1)\\\lambda_1=1\quad\lambda_{2,3}=\frac{1\pm\sqrt{5}}{2}\end{cases}$

$(n_0,n_+,n_-)=(0,2,1)\rightsquigarrow$ NON DEF.

CQ: $x^2+(y-z)^2-z^2\rightsquigarrow q(v)\geq 0\ v=(5,5,0)\quad q(v)\leq 0\ v=(0,0,5)$

SY: $(2,1,3)$ $+\overset{P}{+}\overset{P}{+}\overset{V}{-}$ $n_+=2$ $n_-=1$ CAR: $-\lambda^3+2\lambda^2-2$ $n_0=0$ $n_+=2$ $n_-=3-2=1$

(f) $x^2+y^2+3yz \rightsquigarrow \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 3/2 \\ 0 & 3/2 & 0 \end{pmatrix}$ $|A-\lambda I| = \begin{vmatrix} 1-\lambda & 0 & 0 \\ 0 & 1-\lambda & 3/2 \\ 0 & 3/2 & -\lambda \end{vmatrix} = \begin{cases} -\lambda(1-\lambda)^2 - 9/4\,(1-\lambda) = \\ = (1-\lambda)(\lambda^2-\lambda-9/4) = 0 \\ \lambda_1 = 1 \quad \lambda_{2,3} = \frac{1 \pm \sqrt{10}}{2} \end{cases}$

$(m_0, m_+, m_-) = (0, 2, 1) \rightsquigarrow$ NON DEF.

CQ: $x^2 + (y+\frac{3}{2}z)^2 - \frac{9}{4}z^2 \rightsquigarrow q(v) \geq 0 \; v=(\delta,\delta,0) \quad q(v) \leq 0 \; v=(0,0,\delta)$

SY: $(1,2,3)$ $+ + + -$ $m_+=2$ $m_-=1$ CAR: $-\lambda^3+2\lambda^2+\frac{5}{4}\lambda-9/4$ $m_0=0$ $m_+=2$ $m_-=3-2=1$

(g) $x^2-y^2-3yz \rightsquigarrow \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & -3/2 \\ 0 & -3/2 & 0 \end{pmatrix}$ $|A-\lambda I| = \begin{vmatrix} 1-\lambda & 0 & 0 \\ 0 & -1-\lambda & -3/2 \\ 0 & -3/2 & -\lambda \end{vmatrix} = \begin{cases} \lambda(1-\lambda^2) - 9/4\,(1-\lambda) = \\ = (1-\lambda)(\lambda^2+\lambda-9/4) = 0 \\ \lambda_1 = 1 \quad \lambda_{2,3} = \frac{-1 \pm \sqrt{10}}{2} \end{cases}$

$(m_0, m_+, m_-) = (0, 2, 1) \rightsquigarrow$ NON DEF.

CQ: $x^2 - (y+\frac{3}{2}z)^2 + \frac{9}{4}z^2 \rightsquigarrow q(v) \geq 0 \; v=(\delta,0,\delta) \quad q(v) \leq 0 \; v=(0,\delta,0)$

SY: $(1,2,3)$ $+ + - -$ $(2,1,3)$ $+ - - -$ $m_+=2$ $m_-=1$ CAR: $-\lambda^3+\frac{13}{4}\lambda-9/4$ $m_0=0$ $m_+=2$ $m_-=3-2=1$

(h) $z^2+3yz+5xz \rightsquigarrow \begin{pmatrix} 0 & 0 & 5/2 \\ 0 & 0 & 3/2 \\ 5/2 & 3/2 & 1 \end{pmatrix}$ $|A-\lambda I| = \begin{vmatrix} -\lambda & 0 & 5/2 \\ 0 & -\lambda & 3/2 \\ 5/2 & 3/2 & 1-\lambda \end{vmatrix} = \begin{cases} \lambda^2(1-\lambda) + \frac{5}{2}\lambda + \frac{3}{2}\lambda = \\ = \lambda(-\lambda^2+\lambda+4) = 0 \\ \lambda_1 = 0 \quad \lambda_{2,3} = \frac{1 \pm \sqrt{17}}{2} \end{cases}$

$(m_0, m_+, m_-) = (1, 1, 1) \rightsquigarrow$ NON DEF.

CQ: $(z+\frac{3}{2}y+\frac{5}{2}x)^2 - \frac{9}{4}y^2 - \frac{25}{4}x^2 - \frac{15}{2}xy = (z+\frac{3}{2}y+\frac{5}{2}x)^2 - (\frac{5}{2}x+\frac{3}{2}y)^2$

$q(v) \geq 0 \; v=(\frac{2}{5}\delta, \frac{-2}{3}\delta, \delta) \quad q(v) \leq 0 \; v=(\delta, \delta, -4\delta)$

SY: NON APPLICABILE (DET A=0) CAR: $-\lambda^3+\lambda^2+4\lambda$ $m_0=1$ $m_+=1$ $m_-=3-1-1=1$

(i) $2xy-y^2-z^2 \rightsquigarrow \begin{pmatrix} 0 & 1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$ $|A-\lambda I| = \begin{vmatrix} -\lambda & 1 & 0 \\ 1 & -1-\lambda & 0 \\ 0 & 0 & -1-\lambda \end{vmatrix} = \begin{cases} -\lambda(1+\lambda)^2 + (1+\lambda) = \\ = (1+\lambda)(-\lambda^2-\lambda+1) = 0 \\ \lambda_1 = -1 \quad \lambda_{2,3} = \frac{-1 \pm \sqrt{5}}{2} \end{cases}$

$(m_0, m_+, m_-) = (0, 1, 2) \rightsquigarrow$ NON DEF.

CQ: $-(x-y)^2 + x^2 - z^2 \rightsquigarrow q(v) \geq 0 \; v=(\delta,\delta,0) \quad q(v) \leq 0 \; v=(0,\delta,\delta)$

SY: $(2,3,1)$ $+ - + +$ $m_+=1$ $m_-=2$ CAR: $-\lambda^3-2\lambda^2+1$ $m_0=0$ $m_+=1$ $m_-=3-1=2$

(l) $x^2+2y^2+3z^2+4xy+5xz+6yz \rightsquigarrow \begin{pmatrix}1&2&5/2\\2&2&3\\5/2&3&3\end{pmatrix}$ $\quad |A-\lambda I| = \begin{vmatrix}1-\lambda&2&5/2\\2&2-\lambda&3\\5/2&3&3-\lambda\end{vmatrix}$

$\rightsquigarrow \begin{cases}(1-\lambda)(\lambda^2-5\lambda+6)+15+15-\frac{25}{4}(2-\lambda)-9(1-\lambda)-4(3-\lambda) = -\lambda^3+5\lambda^2-6\lambda+\lambda^2-5\lambda+6+30+\\ -\frac{25}{2}+\frac{25}{4}\lambda-9+9\lambda-12+4\lambda = -\lambda^3+6\lambda^2+\frac{33}{4}\lambda+\frac{5}{2} = 0 \rightsquigarrow 2\lambda^3-12\lambda^2-\frac{33}{2}\lambda-5=0\end{cases}$

($+\frac{5}{2}$: DET(A))

$\begin{cases}\lambda=\frac{m}{n} \quad m=1,5 \quad n=1,2 \rightsquigarrow \lambda = \pm1 \ \pm\frac12 \ \pm\frac52 \ \pm5\\ p(1)=2-12-33/2-5<0 \quad p(-1)=-2-12+33/2-5<0\\ p(1/2)=1/4-3-33/4-5<0 \quad p(-1/2)=-1/4-3+33/4-5=\frac{-1-12+33-20}{4}=0\end{cases}$

	2	−12	−33/2	−5
−1/2		−1	13/2	+5
	2	−13	−20/2	=

$\rightsquigarrow p(\lambda)=(\lambda+1/2)(2\lambda^2-13\lambda-10)=0$

$\lambda_1=-1/2 \quad \lambda_{2,3}=\frac{13\pm\sqrt{169+80}}{4}=\frac{13\pm\sqrt{249}}{4}$

(n_0, n_+, n_-) $(0,1,2)$ NON DEF.

CQ: $x^2+2y^2+3z^2+4xy+5xz+6yz = (x+2y+\frac52 z)^2-4y^2-\frac{25}{4}z^2-10yz+2y^2+3z^2+6yz = (x+2y+\frac52 z)^2-2y^2-\frac{13}{4}z^2-4yz = (x+2y+\frac52 z)^2-2(y+z)^2+2z^2-\frac{13}{4}z^2 = (x+2y+\frac52 z)^2-2(y+z)^2-\frac54 z^2$

VER. $x^2+4y^2+\frac{25}{4}z^2+4xy+5xz+10yz-2y^2-2z^2-4yz-\frac54 z^2 =$
$= x^2+2y^2+3z^2+4xy+5xz+6yz$

$\rightsquigarrow p(v)\geq 0 \quad v=(\delta,0,0) \quad p(v)\leq 0 \quad v=(-2\delta-\frac52\delta, \delta, \delta)$

SY: $(1,2,3)$ $+ + - +$ (P V V) $n_+=1$ $n_-=2$ CAR: $-\lambda^3+6\lambda^2+\frac{33}{4}\lambda+\frac52$ (V P P) $n_0=0$ $n_+=1$ $n_-=3-1=2$

(m) $x^2-2y^2+3z^2+4xy+5xz+6yz \rightsquigarrow \begin{pmatrix}1&2&5/2\\2&-2&3\\5/2&3&3\end{pmatrix}$ $\quad DET = -6+15+15+\frac{25}{2}-9-12 = 3+\frac{25}{2}=\frac{31}{2}$

CQ: $q(v) = (x+2y+\frac52 z)^2-4y^2-\frac{25}{4}z^2-10yz-2y^2+3z^2+6yz =$
$= (x+2y+\frac52 z)^2-6y^2-\frac{13}{4}z^2-4yz = (x+2y+\frac52 z)^2-6(y+\frac13 z)^2+$
$+\frac23 z^2-\frac{13}{4}z^2 = (x+2y+\frac52 z)^2-6(y+\frac13 z)^2-\frac{31}{12}z^2$

(n_0, n_+, n_-) $(0,1,2)$ NON DEF.

VER. $x^2+4y^2+\frac{25}{4}z^2+4xy+5xz+10yz-6y^2-\frac23 z^2-4yz-\frac{31}{12}z^2 =$
$= x^2-2y^2+\frac{75\cancel{-8}^{\,3}-31}{12}z^2+4xy+5xz+6yz$

$\leadsto$ $q(v) \geq 0$ $v=(\delta,0,0)$ $q(v) \leq 0$ $v=(-2S-\frac{5}{2}\delta, S, \delta)$

SY: $(2,1,3)$ $+ - - +$ (V P V) $m_+=$ $m_-=2$ CAR: —

(m) $x^2+y^2+z^2+2xy+2xz+2yz \leadsto \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$ $|A-\lambda I| = \begin{vmatrix} 1-\lambda & 1 & 1 \\ 1 & 1-\lambda & 1 \\ 1 & 1 & 1-\lambda \end{vmatrix} =$

$$\begin{cases} (1-\lambda)^3+1+1-3(1-\lambda) = -\lambda^3+3\lambda^2-3\lambda+1+1-1+3\lambda = \\ = \lambda^2(3-\lambda)=0 \leadsto \lambda_1=\lambda_2=0 \quad \lambda_3=3 \end{cases}$$

m_0 m_+ m_-
$(2,1,0)$
SEMIDEF. POS.

CQ: $q(v)=(x+y+z)^2$ $m_+=1$ $m_0=2$ $q(v)>0$ $v=(\delta,\delta,\delta)$ $\delta \neq 0$

SY: NON APPLICABILE CAR: $-\lambda^3+3\lambda^2$ (V) $m_0=2$ $m_+=1$ $m_-=3-2-1=0$

(o) $x^2-y^2+z^2+2xy+2yz+2xz \leadsto \begin{pmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$ $|A-\lambda I| = \begin{vmatrix} 1-\lambda & 1 & 1 \\ 1 & -1-\lambda & 1 \\ 1 & 1 & 1-\lambda \end{vmatrix} =$

$$\begin{cases} -(\lambda+1)(\lambda^2-2\lambda+1)+1+1+(\lambda+1)-2(1-\lambda) = -\lambda^3+2\lambda^2-\lambda-\lambda^2+2\lambda-1+1+\lambda+1-2+2\lambda = \\ = -\lambda^3+\lambda^2+4\lambda = -\lambda(\lambda^2-\lambda-4)=0 \quad \lambda_1=0 \quad \lambda_{2,3}=\frac{1\pm\sqrt{17}}{2} \end{cases}$$

m_0 m_+ m_-
$(1,1,1)$
NON DEF.

CQ: $(x+y+z)^2-2y^2$ $m_0=1$ $m_+=1$ $m_-=1$

$q(v) \geq 0$ $v=(\delta,0,\delta)$ $q(v) \leq 0$ $v=(-\delta/2, \delta, -\delta/2)$

SY: NON APPLICABILE CAR: $-\lambda^3+\lambda^2+4\lambda$ (V P) $m_0=1$ $m_+=1$ $m_-=3-1-1=1$

(p) $x^2+y^2+z^2+2xy+2yz-2xz \leadsto \begin{pmatrix} 1 & 1 & -1 \\ 1 & 1 & 1 \\ -1 & 1 & 1 \end{pmatrix}$ $|A-\lambda I| = \begin{vmatrix} 1-\lambda & 1 & -1 \\ 1 & 1-\lambda & 1 \\ -1 & 1 & 1-\lambda \end{vmatrix} =$

	-1	+3	0	-4
2		-2	2	
	-1	1	2	=

$(1-\lambda)^3-1-1-3(1-\lambda) = 1-3\lambda+3\lambda^2-\lambda^3-2-3+3\lambda =$

$= -\lambda^3+3\lambda^2-4 = (\lambda-2)(-\lambda^2+\lambda+2)=0$ $\lambda_1=2$ $\lambda_{2,3}=\frac{1\pm\sqrt{9}}{2}=\begin{cases}2\\-1\end{cases}$

m_0 m_+ m_-
$(0,2,1)$
NON DEF.

CQ: $q(v)=(x+y+z)^2-4xz=(x+y+z)^2+(x+z)^2-(x-z)^2$ $m_+=2$ $m_-=1$

$q(v) \geq 0$ $v=(\delta,S,\delta)$ $q(v) \leq 0$ $v=(\delta,0,-\delta)$

SY: NON APPLICABILE CAR: $-\lambda^3+3\lambda^2-4$ (V V) $m_0=0$ $m_+=2$ $m_-=3-2=1$

(p) $x^2+y^2+z^2+2xy+2yz+3xz \rightsquigarrow \begin{pmatrix} 1 & 1 & 3/2 \\ 1 & 1 & 1 \\ 3/2 & 1 & 1 \end{pmatrix}$ $|A-\lambda I| = \begin{vmatrix} 1-\lambda & 1 & 3/2 \\ 1 & 1-\lambda & 1 \\ 3/2 & 1 & 1-\lambda \end{vmatrix} =$

$(1-\lambda)^3+3-\frac{17}{5}(1-\lambda) = 1-3\lambda+3\lambda^2-\lambda^3-\frac{17}{5}+\frac{17}{5}\lambda = -\lambda^3+3\lambda^2+\frac{5}{5}\lambda-\frac{13}{5} =$

	-1	3	5/5	-13/5
1		-1	2	13/5
	-1	2	13/5	=

$= (\lambda-1)(-\lambda^2+2\lambda+13/5)=0 \quad \lambda_1=1 \quad \lambda_2=\frac{-5\pm\sqrt{17}}{2}$

$m_0\ m_+\ m_-$
$(0,2,1)$

NON DEF.

CQ: $q(v)=(x+y+z)^2+xz = (x+y+z)^2+\frac{1}{5}(x+z)^2-\frac{1}{5}(x-z)^2$

$q(v)\geq 0 \quad v=(\delta,5,\delta) \qquad q(v)\leq 0 \quad v=(\delta,0,-\delta)$

SY: $(1,3,2)$ $+\,+\,-\,-$ (P V P) $\quad m_+=2 \ m_-=2 \qquad$ CAR: $-\lambda^3+3\lambda^2+\frac{5}{5}\lambda-\frac{13}{5}$ (V P V) $\quad m_0=0 \ m_+=2 \ m_-=3-2=1$

(2) $x^2+y^2+z^2+xy+2yz+3xz \rightsquigarrow \begin{pmatrix} 1 & 1/2 & 3/2 \\ 1/2 & 1 & 1 \\ 3/2 & 1 & 1 \end{pmatrix}$ $|A-\lambda I| = \begin{vmatrix} 1-\lambda & 1/2 & 3/2 \\ 1/2 & 1-\lambda & 1 \\ 3/2 & 1 & 1-\lambda \end{vmatrix} =$

$(1-\lambda)^3+3/2-3(1-\lambda) = 1-3\lambda+3\lambda^2-\lambda^3+3/2-3+3\lambda = -\lambda^3+3\lambda^2-1/2$

CQ: $(x+\frac{y}{2}+\frac{3}{2}z)^2-\frac{y^2}{5}-\frac{9}{5}z^2-\frac{3}{2}yz+y^2+z^2+2yz = (x+\frac{y}{2}+\frac{3}{2}z)^2+\frac{3y^2}{5}-\frac{5}{5}z^2+\frac{yz}{2} =$

$=(x+\frac{y}{2}+\frac{3}{2}z)^2+\frac{3}{5}(y+\frac{z}{3})^2-\frac{z^2}{12}-\frac{5}{5}z^2 = (x+\frac{y}{2}+\frac{3}{2}z)^2+\frac{3}{5}(y+\frac{z}{3})^2-\frac{5}{3}z^2$

VER. $x^2+\frac{y^2}{5}+\frac{9}{5}z^2+xy+3xz+\frac{3}{2}yz+\frac{3}{5}y^2+\frac{z^2}{12}+\frac{yz}{2}-\frac{5}{3}z^2$

$x^2+y^2+\frac{27+1-16}{12}z^2+xy+3xz+2yz$

$m_0\ m_+\ m_-$
$(0,2,1)$

NON DEF.

$q(v)\geq 0 \quad v=(\delta,5,0) \qquad q(v)\leq 0 \quad v=(\frac{-5\delta}{3},-\frac{\delta}{3},\delta)$

SY: $(1,2,3)$ $+\,+\,+\,-$ (P P V) $\quad m_+=2 \ m_-=1 \qquad$ CAR: $-\lambda^3+3\lambda^2-1/2$ (V V) $\quad m_0=0 \ m_+=2 \ m_-=3-2=1$

(5) $xy-2yz+3xz \rightsquigarrow \begin{pmatrix} 0 & 1/2 & 3/2 \\ 1/2 & 0 & -1 \\ 3/2 & -1 & 0 \end{pmatrix}$ $|A-\lambda I| = \begin{vmatrix} -\lambda & 1/2 & 3/2 \\ 1/2 & -\lambda & -1 \\ 3/2 & -1 & -\lambda \end{vmatrix}$ $= -\lambda^3-3/2+7/2\lambda = -\lambda^3+\frac{7}{2}\lambda-\frac{3}{2}$

CQ: $(ax+by+cz)^2-(ax-by+dz)^2+(d^2-c^2)z^2$

$$\begin{cases} 5ab=1 \\ 2ac-2ad=3 \\ 2bc+2bd=-2 \end{cases} \quad \begin{cases} a=1/2 \quad b=1/2 \\ c-d=3 \\ c+d=-2 \end{cases} \quad \begin{cases} a=1/2 \quad b=1/2 \\ c=1/2 \\ d=-5/2 \end{cases}$$

$q(v) = \left(\frac{x}{2}+\frac{y}{2}+\frac{z}{2}\right)^2 - \left(\frac{x}{2}-\frac{y}{2}-\frac{5}{2}z\right)^2 + 6z^2$ $\quad$ (m_0, m_+, m_-) $(0,2,1)$ NON DEF.

VER. $\frac{x^2}{4}+\frac{y^2}{4}+\frac{z^2}{4}+\frac{xy}{2}+\frac{xz}{2}+\frac{yz}{2}-\frac{x^2}{4}-\frac{y^2}{4}-\frac{25}{4}z^2+\frac{xy}{2}+\frac{5}{2}xz-\frac{5}{2}yz+6z^2$

$xy+3xz-2yz$

$q(v)\geq 0 \quad v=(\delta, \delta-5\delta, \delta) \qquad q(v)\leq 0 \quad v=(\delta,-\delta,0)$

SY: NON APPLICABILE $\quad$ CAR: $-\lambda^3+\frac{7}{2}\lambda-\frac{3}{2}$ $\quad m_0=0 \; m_+=2 \; m_-=3-2=1$

(δ) $-y^2+z^2+xy-yz \rightsquigarrow \begin{pmatrix} 0 & 1/2 & 0 \\ 1/2 & -1 & -1/2 \\ 0 & -1/2 & 1 \end{pmatrix} \quad |A-\lambda I| = \begin{vmatrix} -\lambda & 1/2 & 0 \\ 1/2 & -1-\lambda & -1/2 \\ 0 & -1/2 & 1-\lambda \end{vmatrix} =$

$\lambda(1-\lambda^2)+\frac{1}{4}\lambda-\frac{1}{4}(1-\lambda) = -\lambda^3+\lambda+\frac{1}{4}\lambda-\frac{1}{4}+\frac{1}{4}\lambda = -\lambda^3+\frac{3}{2}\lambda-\frac{1}{4}$

CQ: $-\left(-\frac{x}{2}+y+\frac{z}{2}\right)^2+\frac{x^2}{4}+\frac{z^2}{4}-\frac{xz}{2}+z^2 = -\left(-\frac{x}{2}+y+\frac{z}{2}\right)^2+\frac{x^2}{4}+\frac{5}{4}z^2-\frac{xz}{2} =$

$= -\left(-\frac{x}{2}+y+\frac{z}{2}\right)^2+\frac{1}{4}(x-z)^2-\frac{z^2}{4}+\frac{5}{4}z^2 = -\left(-\frac{x}{2}+y+\frac{z}{2}\right)^2+\frac{1}{4}(x-z)^2+z^2$

VER. $-\frac{x^2}{4}-y^2-\frac{z^2}{4}+xy+\frac{xz}{2}-yz+\frac{x^2}{4}+\frac{z^2}{4}-\frac{xz}{2}+z^2$

$q(v)\geq 0 \quad v=(\delta, \frac{\delta}{2}-\frac{\delta}{2}, \delta) \qquad q(v)\leq 0 \quad v=(0,\delta,0)$

SY: $(3,2,2)$ $+ + - -$ $\quad m_+=2 \; m_-=1$ $\quad$ CAR: $-\lambda^3+\frac{3}{2}\lambda-\frac{1}{4}$ $\quad m_0=0 \; m_+=2 \; m_-=3-2=1$

(μ) $y^2+7xz \rightsquigarrow \begin{pmatrix} 0 & 0 & 7/2 \\ 0 & 1 & 0 \\ 7/2 & 0 & 0 \end{pmatrix} \quad |A-\lambda I| = \begin{vmatrix} -\lambda & 0 & 7/2 \\ 0 & 1-\lambda & 0 \\ 7/2 & 0 & -\lambda \end{vmatrix} =$

$\lambda^2(1-\lambda)-\frac{49}{4}(1-\lambda) =$

$=(1-\lambda)(\lambda^2-49/4)$

$\lambda_1=1 \quad \lambda_{2,3}=\pm 7/2$

(m_0, m_+, m_-) $(0,2,1)$ NON DEF.

CQ: $y^2+\frac{7}{4}(x+z)^2-\frac{7}{4}(x-z)^2 \quad m_+=2 \; m_-=1$

$q(v)\geq 0 \quad v=(\delta,\delta,\delta) \qquad q(v)\leq 0 \quad v=(\delta,0,-\delta)$

SY: NON APPLICABILE $\quad$ CAR: $(1-\lambda)(\lambda^2-49/4) = -\lambda^3+\lambda^2+\frac{49}{4}\lambda-\frac{49}{4}$

$m_0=0 \quad m_+=2 \quad m_-=3-2=1$

3. Forme quadratiche $q(x,y,z,w)$ in $\mathbb{R}^4$:

(a) $x^2 - w^2$ (b) $x^2 - 2y^2 + 3z^2 - 4w^2$ (c) $y^2 - 3z^2 - 5w^2$

(d) $x^2 + yz - w^2$ (e) $x^2 + y^2 + w^2 + zx$ (f) $-x^2 + z^2 + w^2 - zy$

(g) $-y^2 - w^2 - 3xz$ (h) $2y^2 + 3z^2 - 5w^2 + xz - 3yw$ (i) $xy - 3zw$

(l) $x^2 + y^2 - z^2 - w^2 - 2xy - 2zw$ (m) $x^2 + y^2 - z^2 - w^2 - 2xz - 2yw$

(a) $x^2 - w^2 \rightsquigarrow \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$ $|A-\lambda I| = \begin{vmatrix} 1-\lambda & 0 & 0 & 0 \\ 0 & -\lambda & 0 & 0 \\ 0 & 0 & -\lambda & 0 \\ 0 & 0 & 0 & -1-\lambda \end{vmatrix} = \begin{cases} \lambda^2(\lambda^2-1)=0 \\ \lambda_1=\lambda_2=0 \\ \lambda_{3,4}=\pm 1 \end{cases}$

$m_0\ m_+\ m_-$: $(2, 1, 1)$ NON DEF.

CQ: $x^2 - w^2$ $m_+=1$ $m_-=1$ $m_0=2$

$q(v)\geq 0$ $v(\delta,0,0,0)$ $q(v)\leq 0$ $v=(0,0,0,\delta)$

SY: N.A. CAR: $\lambda^4 - \lambda^2$ $m_0=2$ $m_+=1$ $m_-=4-2-1=1$

(b) $x^2 - 2y^2 + 3z^2 - 5w^2 \rightsquigarrow \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & -5 \end{pmatrix}$ $|A-\lambda I| = (1-\lambda)(-2-\lambda)(3-\lambda)(-5-\lambda)=0$

$\lambda_1=1$ $\lambda_2=-2$ $\lambda_3=3$ $\lambda_4=-5$

$m_0\ m_+\ m_-$: $(0, 2, 2)$ NON DEF.

CQ: $x^2 - 2y^2 + 3z^2 - 5w^2$ $m_+=2$ $m_-=2$

$q(v)\geq 0$ $v=(\delta,0,5,0)$ $q(v)\leq 0$ $v=(0,\delta,0,5)$

SY: $(1,2,3,5)$ $+\ +\ -\ -\ +$ (P V P V) $m_+=2$ $m_-=2$

CAR: $(-2-\lambda+2\lambda+\lambda^2)(-12-3\lambda+4\lambda+\lambda^2) = (\lambda^2+\lambda-2)(\lambda^2+\lambda-12) =$

$= \lambda^4+\lambda^3-12\lambda^2+\lambda^3+\lambda^2-12\lambda-2\lambda^2-2\lambda+24 = \lambda^4+2\lambda^3-13\lambda^2-14\lambda+24$ (P V P V)

$\rightsquigarrow m_0=0$ $m_+=2$ $m_-=4-2=2$

(c) $y^2 - 3z^2 - 5w^2 \rightsquigarrow \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -3 & 0 \\ 0 & 0 & 0 & -5 \end{pmatrix}$ $|A-\lambda I| = -\lambda(1-\lambda)(-3-\lambda)(-5-\lambda)$

$\lambda_1=0$ $\lambda_2=1$ $\lambda_3=-3$ $\lambda_4=-5$

$m_0\ m_+\ m_-$: $(1, 1, 2)$ NON DEF.

CQ: $y^2 - 3z^2 - 5w^2$ $m_+=1$ $m_-=2$ $m_0=1$

$q(v)\geq 0$ $v=(\delta,\delta,0,0)$ $q(v)\leq 0$ $v=(0,0,\delta,5)$

SY: N.A. CAR: $(\lambda^2-\lambda)(\lambda^2+8\lambda+15) = \lambda^4+8\lambda^3+15\lambda^2-\lambda^3-8\lambda^2-15\lambda =$

$= \lambda^4 \overset{P}{+} 7\lambda^3 \overset{P}{+} 7\lambda^2 \overset{V}{-} 15\lambda \quad m_0=1 \quad m_+=1 \quad m_-=4-1-1=2$

(d) $x^2+yz-w^2 \rightsquigarrow \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1/2 & 0 \\ 0 & 1/2 & 0 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \quad |A-\lambda I| = \begin{vmatrix} 1-\lambda & 0 & 0 & 0 \\ 0 & -\lambda & 1/2 & 0 \\ 0 & 1/2 & -\lambda & 0 \\ 0 & 0 & 0 & -1-\lambda \end{vmatrix} =$

$= (1-\lambda)[\lambda^2(-1-\lambda)+1/4(1+\lambda)] = (1-\lambda)(1+\lambda)(-\lambda^2+1/4)=0$

$\lambda_1=1 \quad \lambda_2=-1 \quad \lambda_{3,4}=\pm 1/2 \qquad (m_0, m_+, m_-) = (0,2,2)$ NON DEF.

CQ: $x^2+\frac{1}{4}(y+z)^2-\frac{1}{4}(y-z)^2-w^2 \quad m_+=2 \quad m_-=2$

$q(v)\geq 0 \quad v=(\delta, s, s, 0) \qquad q(v)\leq 0 \quad v=(0,\delta,-\delta,s)$

SY: N.A. CAR: $(1-\lambda^2)(-\lambda^2+1/4) = \lambda^4+1/4\lambda^2-\lambda^2+1/4 =$

$= \lambda^4 \overset{V}{-} 3/4\,\lambda^2 \overset{V}{+} 1/4 \quad m_0=0 \quad m_+=2 \quad m_-=4-2=2$

(e) $x^2+y^2+w^2+zx \rightsquigarrow \begin{pmatrix} 1 & 0 & 1/2 & 0 \\ 0 & 1 & 0 & 0 \\ 1/2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad |A-\lambda I| = \begin{vmatrix} 1-\lambda & 0 & 1/2 & 0 \\ 0 & 1-\lambda & 0 & 0 \\ 1/2 & 0 & -\lambda & 0 \\ 0 & 0 & 0 & 1-\lambda \end{vmatrix} =$

$= (1-\lambda)[-\lambda(1-\lambda)^2-1/4(1-\lambda)] = (1-\lambda)^2(\lambda^2-\lambda-1/4)=0$

$\lambda_{1,2}=1 \quad \lambda_{3,4}=\frac{1\pm\sqrt{2}}{2} \qquad (m_0,m_+,m_-)=(0,3,1)$ NON DEF.

CQ: $(x+\frac{z}{2})^2-\frac{z^2}{4}+y^2+w^2 \quad m_+=3 \quad m_-=1$

$q(v)\geq 0 \quad v=(\delta, s, 0, r) \qquad q(v)\leq 0 \quad v=(-\delta/2, 0, \delta, 0)$

SY: $(1,2,3,4)$ $+ \overset{P}{} + \overset{P}{} + \overset{V}{} - \overset{P}{} -$ $\quad m_+=3 \quad m_-=1$

CAR: $(\lambda^2-2\lambda+1)(\lambda^2-\lambda-1/4) = \lambda^4-\lambda^3-1/4\lambda^2-2\lambda^3+2\lambda^2+1/2\lambda+\lambda^2-\lambda-1/4=$

$= \lambda^4 \overset{V}{-} 3\lambda^3 \overset{V}{+} 11/4\,\lambda^2 \overset{V}{-} 1/2\,\lambda \overset{P}{-} 1/4 \quad m_0=0 \quad m_+=3 \quad m_-=4-3=1$

(f) $-x^2+z^2+w^2-zy \rightsquigarrow \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 0 & -1/2 & 0 \\ 0 & -1/2 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$ $|A-\lambda I| = \begin{vmatrix} -1-\lambda & 0 & 0 & 0 \\ 0 & -\lambda & -1/2 & 0 \\ 0 & -1/2 & 1-\lambda & 0 \\ 0 & 0 & 0 & 1-\lambda \end{vmatrix} =$

$= (-1-\lambda)[-\lambda(1-\lambda)^2 - 1/4(1-\lambda)] = (-1-\lambda)(1-\lambda)(\lambda^2-\lambda-1/4) = 0$

$\lambda_1 = -1 \quad \lambda_2 = 1 \quad \lambda_{3,4} = \frac{1 \pm \sqrt{2}}{2}$ $\quad (m_0, m_+, m_-) = (0,2,2)$ NON DEF.

CQ: $-x^2+w^2+(z-y/2)^2 - y^2/4 \quad m_+=2 \quad m_-=2$

$q(v) \geq 0 \quad v=(0,0,\delta,\delta) \qquad q(v) \leq 0 \quad v=(\delta,\delta,\delta/2,0)$

SY: $(3,2,2,4)$ $+ + + - +$ (P P V V) $m_+=2 \quad m_-=2$

CAR: $(\lambda^2-1)(\lambda^2-\lambda-1/4) = \lambda^4-\lambda^3-1/4\lambda^2-\lambda^2+\lambda+1/4 = \lambda^4-\lambda^3-5/4\lambda^2+\lambda+1/4$ (V P V P)

$m_0=0 \quad m_+=2 \quad m_-=4-2=2$

(g) $-y^2-w^2-3xz \rightsquigarrow \begin{pmatrix} 0 & 0 & -3/2 & 0 \\ 0 & -1 & 0 & 0 \\ -3/2 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$ $|A-\lambda I| = \begin{vmatrix} -\lambda & 0 & -3/2 & 0 \\ 0 & -1-\lambda & 0 & 0 \\ -3/2 & 0 & -\lambda & 0 \\ 0 & 0 & 0 & -1-\lambda \end{vmatrix} =$

$= (-1-\lambda)[\lambda^2(-1-\lambda) - 9/4(-1-\lambda)] = (1+\lambda)^2(\lambda^2-9/4) = 0$

$\lambda_1=\lambda_2=-1 \quad \lambda_{3,4} = \pm 3/2$ $\quad (m_0, m_+, m_-) = (0,1,3)$ NON DEF.

CQ: $-y^2-w^2-\frac{3}{4}(x+z)^2+\frac{3}{4}(x-z)^2 \quad m_+=1 \quad m_-=3$

$q(v) \geq 0 \quad v=(\delta,0,-\delta,0) \qquad q(v) \leq 0 \quad v=(\delta,\delta,\delta,\delta)$

SY: N.A. CAR: $(\lambda^2+2\lambda+1)(\lambda^2-9/4) = \lambda^4+2\lambda^3+\lambda^2-9/4\lambda^2-9/2\lambda-9/4 =$

$= \lambda^4+2\lambda^3-5/4\lambda^2-9/2\lambda-9/4$ (P V P P) $\quad m_0=0 \quad m_+=1 \quad m_-=4-1=3$

(h) $2y^2+3z^2-5w^2+xz-3yw \rightsquigarrow \begin{pmatrix} 0 & 0 & 1/2 & 0 \\ 0 & 2 & 0 & -3/2 \\ 1/2 & 0 & 3 & 0 \\ 0 & -3/2 & 0 & -5 \end{pmatrix}$ $|A-\lambda I| = \begin{vmatrix} -\lambda & 0 & 1/2 & 0 \\ 0 & 2-\lambda & 0 & -3/2 \\ 1/2 & 0 & 3-\lambda & 0 \\ 0 & -3/2 & 0 & -5-\lambda \end{vmatrix} =$

$= -\lambda[(2-\lambda)(3-\lambda)(-5-\lambda) - 9/4(3-\lambda)] + \frac{1}{2}[9/8 + 1/2(2-\lambda)(5+\lambda)] =$

$$= -\lambda\left[(2-\lambda)(\lambda^2+2\lambda-15) - 27/4 + 9/4\,\lambda\right] + 9/16 + 1/4(-\lambda^2-3\lambda+10) =$$

$$= -\lambda(\cancel{2\lambda^2}+4\lambda-30-\lambda^3-\cancel{2\lambda^2}+15\lambda-27/4+9/4\,\lambda) + 9/16 - \lambda^2/4 - 3/4\,\lambda + 10/4 =$$

$$= -\lambda(-\lambda^3+85/4\,\lambda-147/4) - \lambda^2/4 - 3/4\,\lambda + 49/16 =$$

$$= \lambda^4 - 85/4\,\lambda^2 + 147/4\,\lambda - \lambda^2/4 - 3/4\,\lambda + 49/16 = \lambda^4 - 43/2\,\lambda^2 + 36\lambda + 49/16 = 0$$

CQ: $2y^2+3z^2-5w^2+xz-3yw = 2\left(y-\frac{3}{4}w\right)^2 - \frac{9}{8}w^2 + 3z^2 - 5w^2 + xz =$

$$= 2\left(y-\frac{3}{4}w\right)^2 - \frac{49}{8}w^2 + 3\left(z+\frac{x}{6}\right)^2 - \frac{x^2}{12} \qquad m_+ = 2 \quad m_- = 2$$

VER. $2y^2 + \frac{9}{8}w^2 - 3yw - \frac{49}{8}w^2 + 3z^2 + \cancel{\frac{x^2}{12}} + xz - \cancel{\frac{x^2}{12}} = 2y^2+3z^2-5w^2+xz-3yw$

$q(v) \geq 0 \quad v = (0, \delta, s, 0) \qquad q(v) \leq 0 \quad v = \left(\delta, \frac{3}{4}s, -\frac{\delta}{6}, s\right)$

SY: $(2,3,2,4)$ $+ + + - +$ (P P V V) $\quad m_+ = 2 \quad m_- = 2$

CAR: $\lambda^4 - 43/2\,\lambda^2 + 36\lambda + 49/16$ (V V P) $\quad m_0 = 0 \quad m_+ = 2 \quad m_- = 4-2 = 2$

(i) $xy - 3zw \rightsquigarrow \begin{pmatrix} 0 & 1/2 & 0 & 0 \\ 1/2 & 0 & 0 & 0 \\ 0 & 0 & 0 & -3/2 \\ 0 & 0 & -3/2 & 0 \end{pmatrix} \quad |A-\lambda I| = \begin{vmatrix} -\lambda & 1/2 & 0 & 0 \\ 1/2 & -\lambda & 0 & 0 \\ 0 & 0 & -\lambda & -3/2 \\ 0 & 0 & -3/2 & -\lambda \end{vmatrix} =$

$= (\lambda^2 - 1/4)(\lambda^2 - 9/4) \qquad \lambda_{1,2} = \pm 1/2 \quad \lambda_{3,4} = \pm 3/2 \qquad (m_0, m_+, m_-) = (0, 2, 2)$ NON DEF.

CQ: $\frac{1}{4}(x+y)^2 - \frac{1}{4}(x-y)^2 - \frac{3}{4}(z+w)^2 + \frac{3}{4}(z-w)^2 \qquad m_+ = 2 \quad m_- = 2$

$q(v) \geq 0 \quad v = (\delta, \delta, s, -s) \qquad q(v) \leq 0 \quad v = (\delta, -\delta, s, s)$

SY: N.A. CAR: $(\lambda^2 - 1/4)(\lambda^2 - 9/4) = \lambda^4 - \frac{5}{2}\lambda^2 + \frac{9}{16} \quad m_0 = 0 \quad m_+ = 2 \quad m_- = 4-2 = 2$

(ii) $x^2+y^2-z^2-w^2-2xy-2zw \rightsquigarrow \begin{pmatrix} 1 & -1 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & -1 \\ 0 & 0 & -1 & -1 \end{pmatrix} \quad |A-\lambda I| = \begin{vmatrix} 1-\lambda & -1 & 0 & 0 \\ -1 & 1-\lambda & 0 & 0 \\ 0 & 0 & -1-\lambda & -1 \\ 0 & 0 & -1 & -1-\lambda \end{vmatrix} =$

$$= \left[(1-\lambda)^2 - 1\right]\left[(1+\lambda)^2 - 1\right] = (\lambda^2 - 2\lambda)(\lambda^2 + 2\lambda) = \lambda^2(\lambda-2)(\lambda+2) = 0$$

$\lambda_1=\lambda_2=0 \quad \lambda_3=2 \quad \lambda_4=-2$ $\quad \overset{m_0\ m_+\ m_-}{(2,1,1)}$ NON DEF.

CQ: $(x-y)^2-(z+w)^2 \quad m_+=2 \quad m_-=1$

$q(v)\geq 0 \quad v=(\sigma,0,0,0) \quad q(v)\leq 0 \quad v=(0,0,\sigma,0)$

SY: N.A. CAR: $\lambda^2(\lambda^2-4)=\lambda^4-4\lambda^2 \quad m_0=2 \quad m_+=1 \quad m_-=4-2-1=1$

(m) $x^2+y^2-z^2-w^2-2xz-2yw \rightsquigarrow \begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ -1 & 0 & -1 & 0 \\ 0 & -1 & 0 & -1 \end{pmatrix} \quad |A-\lambda I| = \begin{vmatrix} 1-\lambda & 0 & -1 & 0 \\ 0 & 1-\lambda & 0 & -1 \\ -1 & 0 & -1-\lambda & 0 \\ 0 & -1 & 0 & -1-\lambda \end{vmatrix} =$

$=(1-\lambda)\left[(1-\lambda)(1+\lambda)^2+(1+\lambda)\right]-\left[-1-(1-\lambda^2)\right]=$

$=(1-\lambda)(1+2\lambda+\lambda^2-\lambda-2\lambda^2-\lambda^3+1+\lambda)-(\lambda^2-2)=$

$=(1-\lambda)(-\lambda^3-\lambda^2+2\lambda+2)-\lambda^2+2=-\lambda^3-\lambda^2+2\lambda+2+\lambda^4+\lambda^3-2\lambda^2-2\lambda-\lambda^2+2=$

$=\lambda^4-4\lambda^2+4=(\lambda^2-2)^2 \quad \lambda_1=\lambda_2=+\sqrt{2} \quad \lambda_3=\lambda_4=-\sqrt{2}$ $\quad \overset{m_0\ m_+\ m_-}{(0,2,2)}$ NON DEF.

CQ: $(x-z)^2-2z^2+(y-w)^2-2w^2 \quad m_+=2 \quad m_-=2$

$q(v)\geq 0 \quad v=(\sigma,s,0,0) \quad q(v)\leq 0 \quad v=(\sigma,s,\sigma,s)$

SY: $(1,2,3,4)$ $+ + + - +$ $m_+=2 \quad m_-=2$ CAR: $\lambda^4-4\lambda^2+4 \quad m_0=0 \quad m_+=2 \quad m_-=4-2=2$