Numeri complessi 4

Argomenti: Radici di polinomi **Difficoltà**: ⋆⋆⋆

Prerequisiti: Operazioni tra numeri complessi, radici n-esime

---

Determinare le radici (complesse) dei seguenti polinomi. Nella risposta elencare le radici in ordine crescente di modulo e, a parità di modulo, in ordine crescente di argomento, pensato in $[0, 2\pi)$. Ripetere le radici a seconda della *molteplicità*. Utilizzare la forma cartesiana ovunque possibile, e comunque in tutti i casi in cui l'argomento è un multiplo intero di $\pi/6$ o $\pi/4$.

Polinomio	z_1	z_2	z_3	z_4
x^2+1				
x^4-1				
x^4+1				
x^4+x^2				
x^2+2x+1				
x^4-2x^2+1				
x^4+2x^2+1				
$x^2+4x+13$				
x^2+ix+6				
x^2+x+1				
x^4+x^2+1				
$ix^4+(1+i)x$				
x^4+2x^2+2				
x^3-x+2i				
x^3+x^2+x+1				
$x^4+x^3+x^2+x+1$				

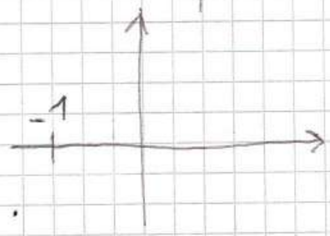
Numeri complessi 4 Test n. 50

Determinare le radici (complesse) dei seguenti polinomi

1) x^2+1 Non ha radici reali - 2 radici complesse

$x^2=-1$ si torna scheda precedente trovare la radice quadrata di -1

$-1 = 1e^{i\pi}$



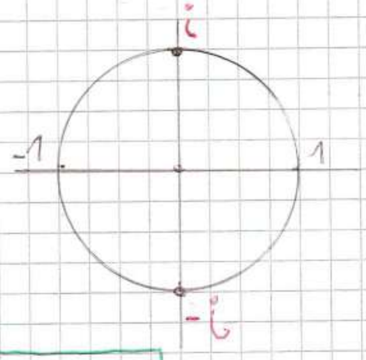
$\rho^2 e^{2\theta i} = e^{\pi i}$

$\rho^2 = 1 \qquad \rho = 1$

$2\theta = \pi + 2k\pi \Rightarrow \theta = \frac{\pi}{2} + k\pi$

$k = 0 \Rightarrow \theta = \frac{\pi}{2}$

$k = 1 \Rightarrow \theta = \frac{3}{2}\pi$



fattorizzazione

$\underbrace{x^2+1} = (x-i)(x+i)$

$= x^2 - \underbrace{i^2}_{-1} = \underbrace{x^2+1}$

Risposte $\boxed{z_1 = i}$ $\boxed{z_2 = -i}$

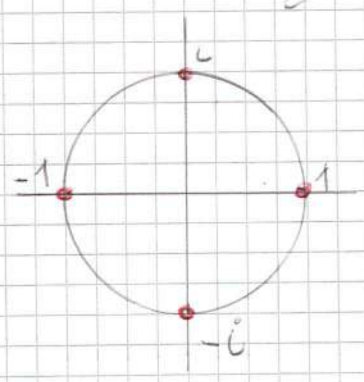
---

2) $x^4 - 1 \Rightarrow x^4 = 1$ trovare la radice quarta di 1

$1 = e^{0i} \qquad \rho^4 e^{4\theta i} = 1e^{0i}$

$\begin{cases} \rho^4 = 1 \\ 4\theta = 0 + 2k\pi \end{cases} \qquad \theta = \frac{k\pi}{2}$

$\theta = 0$; $\theta = \frac{\pi}{2}$; $\theta = \pi$; $\theta = \frac{3}{2}\pi$



Risposte:

$\boxed{z_1 = 1}$; $\boxed{z_2 = i}$; $\boxed{z_3 = -1}$; $\boxed{z_4 = -i}$

3) x^4+1; $x^4=-1$ in forma esponenziale $-1=e^{\pi i}$

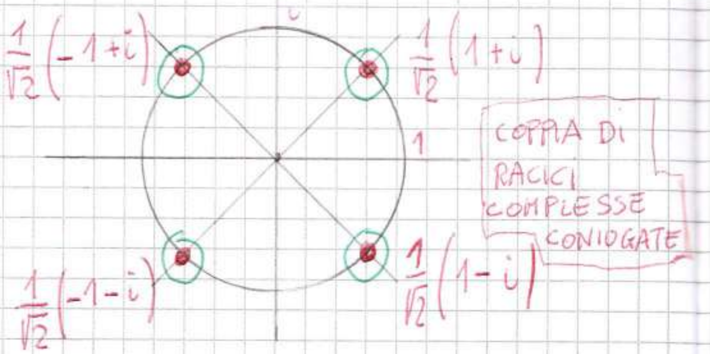
$x^4=\rho^4 e^{4\theta i}$ $\begin{cases} \rho^4=1 \\ 4\theta=\pi+2k\pi \end{cases}$ $\rho=1$ $\theta=\frac{\pi}{4}+\frac{k\pi}{2}$

$k=0 \Rightarrow \theta=\frac{\pi}{4} \Rightarrow \boxed{z_1=\frac{1}{\sqrt{2}}(1+i)}$

$k=1 \Rightarrow \theta=\frac{3\pi}{4} \Rightarrow \boxed{z_2=\frac{1}{\sqrt{2}}(-1+i)}$

$k=2 \Rightarrow \theta=\frac{5\pi}{4} \Rightarrow \boxed{z_3=\frac{1}{\sqrt{2}}(-1-i)}$

$k=3 \Rightarrow \theta=\frac{7\pi}{4} \Rightarrow \boxed{z_4=\frac{1}{\sqrt{2}}(1-i)}$

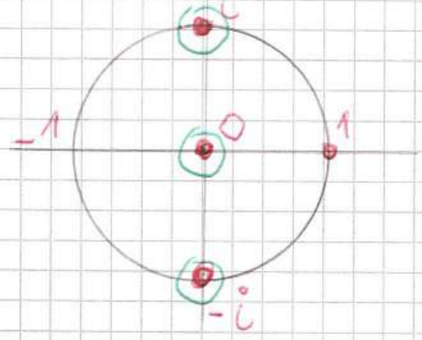


---

4) x^4+x^2 $x^2(x^2+1)=0$ $x^2=0$ radice molteplicità 2

$x^2=-1=e^{\pi i}$

$x^2=\rho^2 e^{i2\theta}$ $\begin{cases} \rho^2=1 & \rho=1 \\ 2\theta=\pi+2k\pi \end{cases}$ $\theta=\frac{\pi}{2}+k\pi$ $\theta_1=\frac{\pi}{2}$, $\theta_2=\frac{3}{2}\pi$



$\boxed{z_1=0}$ $\boxed{z_2=0}$ $\boxed{z_3=i}$ $\boxed{z_4=-i}$

---

5) x^2+2x+1 $\begin{pmatrix} P=1 & S=-2 \end{pmatrix} \Rightarrow x_{12}=-1$

$(x+1)^2$

soluzione reale con $x=-1$ di molteplicità 2

$\boxed{z_1=-1}$ $\boxed{z_2=-1}$

6) $x^4 - 2x^2 + 1 = p(x)$

$x^4 - 2x^2 + 1 = 0 \implies (x^2-1)^2 = (x^2-1)(x^2-1) = (x-1)(x+1)(x-1)(x+1)$

$\boxed{z_1 = 1}$ $\boxed{z_2 = 1}$ $\boxed{z_3 = -1}$ $\boxed{z_4 = -1}$

---

7) $x^4 + 2x^2 + 1 = p(x) \geq 1$ per ogni $x \in \mathbb{R}$

in $\mathbb{C}$ ci devono essere 4 radici se contate con molteplicità

pongo $y = x^2 \implies y^2 + 2y + 1 = 0$ $\quad y_{12} = -1 \pm \sqrt{1-1} = -1$ con molteplicità 2

$x^2 = -1 < \begin{matrix} i \\ -i \end{matrix}$ molteplicità 2

$\boxed{z_1 = i}$ $\boxed{z_2 = i}$ $\boxed{z_3 = -i}$ $\boxed{z_4 = -i}$

---

8) $x^2 + 4x + 13 = 0$ $\quad x_{12} = -2 \pm \sqrt{4-13} = -2 \pm \sqrt{-9}$

$\sqrt{-9} = < \begin{matrix} 3i \\ -3i \end{matrix}$

infatti $-9 = 9e^{\pi i}$

$z^2 = \rho^2 e^{2\theta i}$

$\begin{cases} \rho^2 = 9 & \rho = 3 \\ 2\theta = \pi + k\pi & \theta = \frac{\pi}{2} + k\pi \end{cases}$

$\theta = \frac{\pi}{2}$ e $\theta = \frac{3}{2}\pi$

$z = 3i$ e $-3i$

$\boxed{z_1 = -2 + 3i}$

$\boxed{z_2 = -2 - 3i}$

---

9) $x^2 + ix + 6$ le radici la Somma $= -i$ Prodotto $= 6$

le 2 radici sono $-3i$ ~~e~~ $2i$ $\quad -3i + 2i = -i$

$-3i \cdot 2i = -6i^2 = (-6)(-1) = 6$

$\boxed{z_1 = 2i}$ $\boxed{z_2 = -3i}$

10) x^2+x+1 $\qquad x_{12} = \frac{-1 \pm \sqrt{1-4}}{2} = \frac{-1 \pm \sqrt{-3}}{2}$

$\sqrt{-3} = \begin{cases} i\sqrt{3} \\ -i\sqrt{3} \end{cases}$

radice quadrata di $-3 = 3e^{\pi i}$ argomento diviso per 2

le due radici sono: $\sqrt{3} \cdot e^{\frac{\pi}{2} i} = i\sqrt{3}$

radice del modulo

: $\sqrt{3} \cdot e^{\frac{3}{2}\pi i} = -i\sqrt{3}$

$\frac{\pi}{2} + \frac{2\pi}{2}$

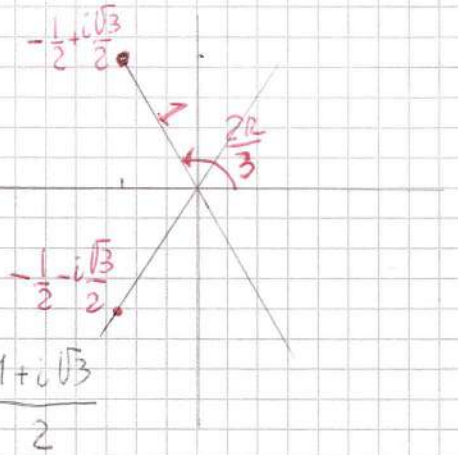
$\boxed{z_1 = \frac{-1+i\sqrt{3}}{2}}$ $\qquad \boxed{z_2 = \frac{-1-i\sqrt{3}}{2}}$

---

11) $x^4 + x^2 + 1 = 0 \quad p(x) \geq 1$ per ogni $x \in \mathbb{R}$

in $\mathbb{C}$ 4 radici pongo $y = x^2 \Rightarrow y^2+y+1=0$ esercizio precedente

$y_{12} = \begin{cases} -\frac{1}{2} + i\frac{\sqrt{3}}{2} \\ -\frac{1}{2} - i\frac{\sqrt{3}}{2} \end{cases}$ devo fare le 2 radici quadrate



1°) $x^2 = -\frac{1}{2} + i\frac{\sqrt{3}}{2} = e^{\frac{2\pi}{3} i}$ (vedi disegno il modulo è $= 1$)

$x_1 = 1 \cdot e^{\frac{\pi}{3} i} = \left(\cos\frac{\pi}{3} + i \sin\frac{\pi}{3}\right) = \frac{1+i\sqrt{3}}{2}$

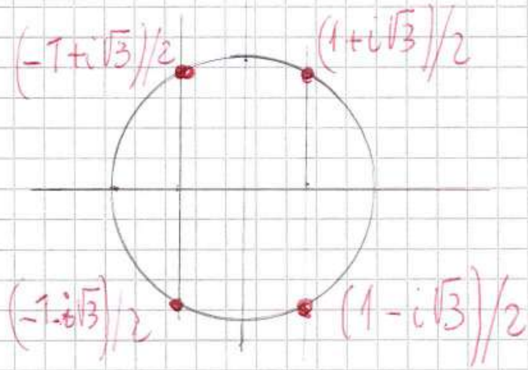
radice del modulo $\qquad$ argomento diviso per 2

$x_2 = 1 \cdot e^{\frac{4}{3}\pi i} = \left(\cos\frac{4}{3}\pi + i \sin\frac{4}{3}\pi\right) = \frac{-1-i\sqrt{3}}{2}$

2°) $x^2 = -\frac{1}{2} - i\frac{\sqrt{3}}{2} = e^{\frac{4}{3}\pi i}$ con $\rho = 1$

$x_3 = e^{\frac{2}{3}\pi i} = \frac{-1+i\sqrt{3}}{2}$ e $x_4 = e^{\frac{5}{3}\pi i} = +\frac{1-i\sqrt{3}}{2}$

$\boxed{z_1 = (1+i\sqrt{3})/2}$, $\boxed{z_2 = (-1+i\sqrt{3})/2}$, $\boxed{z_3 = (-1-i\sqrt{3})/2}$ $\boxed{z_4 = (1-i\sqrt{3})/2}$



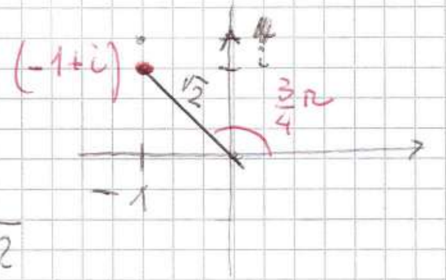
---

12) $i x^4 + (1+i)x = 0 \qquad x\left[i x^3 + (1+i)\right] = 0$

$\boxed{x_1 = 0}$ z_1 $\qquad x^3 = \dfrac{-(1+i)}{i} \cdot \dfrac{i}{i} = \dfrac{-(1+i)\cdot i}{\cancel{-1}} = -1+i$

$x^3 = -1+i$ trovare le radici cubiche di $-1+i$

$-1+i = \sqrt{2}\, e^{\frac{3}{4}\pi i}$ (forma esponenziale e vedi disegno)



il modulo della radice cubica di $\sqrt{2}$ è $= \sqrt[6]{2}$

l'argomento è diviso per 3

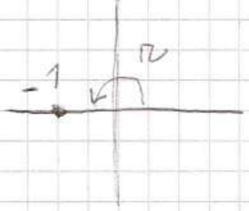
$$x_2 = \sqrt[6]{2}\, e^{\frac{\pi}{4} i} = \sqrt[6]{2}\left(\frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}\right) = \boxed{\frac{1+i}{\sqrt[3]{2}} = x_2} \quad z_2$$

$$x_3 = \sqrt[6]{2} \cdot e^{\left(\frac{\pi}{4} + \frac{2}{3}\pi\right) i} = \boxed{\sqrt[6]{2}\, e^{\frac{11}{12}\pi i} = x_3} \quad z_3$$

$$x_4 = \sqrt[6]{2}\, e^{\left(\frac{\pi}{4} + \frac{4\pi}{3}\right) i} = \boxed{\sqrt[6]{2}\, e^{\frac{19}{12}\pi i} = x_4} \quad z_4$$

13) x^4+2x^2+2 [vedere lezione 106 A.M. I AA. 2012-2013]
l'originale è sempre meglio della copia.

$y=x^2 \Rightarrow y^2+2y+2$



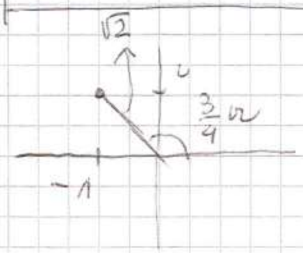
$-1=e^{\pi i}$

le 2 radici quadrate di -1 sono:

$e^{\frac{\pi}{2}i}=i$ e $e^{\frac{3}{2}\pi i}=-i$

$y_{1,2}=-1\pm\sqrt{-1}=-1\pm i$

$x^2=-1\pm i$



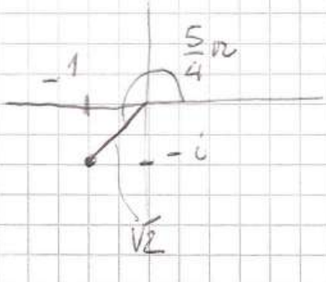
1°) $x^2=-1+i=\sqrt{2}\,e^{\frac{3}{4}\pi i}$

$\boxed{\text{modulo } \sqrt[4]{2} \quad \theta=\frac{3}{8}\pi+k\pi}$

$x_1=\boxed{\sqrt[4]{2}\,e^{\frac{3}{8}\pi i}}$ z_1

$x_2=\sqrt[4]{2}\;e^{\left(\frac{3}{8}\pi+\pi\right)i}=\boxed{\sqrt[4]{2}\,e^{\frac{11}{8}\pi i}}$ z_2

2°) $x^2=-1-i$



$-1-i=\sqrt{2}\,e^{\frac{5}{4}\pi i}$

$x_3=\boxed{\sqrt[4]{2}\;e^{\frac{5}{8}\pi i}}$ z_3 $\qquad x_4=\boxed{\sqrt[4]{2}\,e^{\frac{13}{8}\pi i}}$ z_4

---

14) $x^3-x+2i=0$ si annulla per $x=i$
infatti sostituendo si ha $\underset{\overset{\shortparallel}{i^3}}{-i}-i+2i=0$

si può dividere il polinomio
di partenza per $(x-i)$

$$\begin{array}{l|l} x^3 \quad -x+2i & x-i \\ \hline -x^3+ix^2 & x^2+ix-2 \\ \hline // \quad ix^2-x+2i & \\ \quad -ix^2-x & \\ \hline \quad // \; -2x+2i & \\ \quad\quad +2x-2i & \\ \hline \quad\quad // \quad // & \end{array}$$

$$(x^3-x+2i)=(x-i)(x^2+ix-2)$$

$$x^2+ix-2=0$$

$$x_{1,2}=\frac{-i\pm\sqrt{-1+8}}{2}=\begin{cases}\dfrac{\sqrt{7}-i}{2}\\[2mm] \dfrac{-(\sqrt{7}+i)}{2}\end{cases}$$

$\boxed{z_1=i}$ $\boxed{z_2=-(\sqrt{7}+i)/2}$; $\boxed{z_3=(\sqrt{7}-i)/2}$

---

15) $x^3+x^2+x+1=0$ si annulla per $x=-1$

$$\begin{array}{l|l} x^3+x^2+x+1 & x+1 \\ \hline -x^3-x^2 & x^2+1 \\ \hline // \quad // \quad x+1 & \\ \quad\quad -x-1 & \\ \hline \quad\quad // \quad // & \end{array}$$

$$x^3+x^2+x+1=(x+1)(x^2+1)$$

radici $x+1=0$ $x=-1$

$$x^2=-1=\begin{cases} i\\ -i\end{cases}$$ [Vedi esercizio N. 13]

$\boxed{z_1=i}$; $\boxed{z_2=-1}$ $\boxed{z_3=-i}$

---

16) $x^4+x^3+x^2+x+1=0$ [Vedere lezione A.M.I n° 103 del 2012/13]

$(x^5-1)=(x-1)(x^4+x^3+x^2+x+1)$ le radici cercate sono le radici di $x^5-1=0$ con esclusione di $x-1=0 \Rightarrow x=1$

radici quinte di $x^5-1=0$ $\quad x^5=1=e^{0i}$

$x^5=\rho^5 e^{5\theta i}$ $\quad 5\theta=2k\pi$ $\quad \theta=\frac{2k\pi}{5}$

$\rho^5=1 \Rightarrow \rho=1$

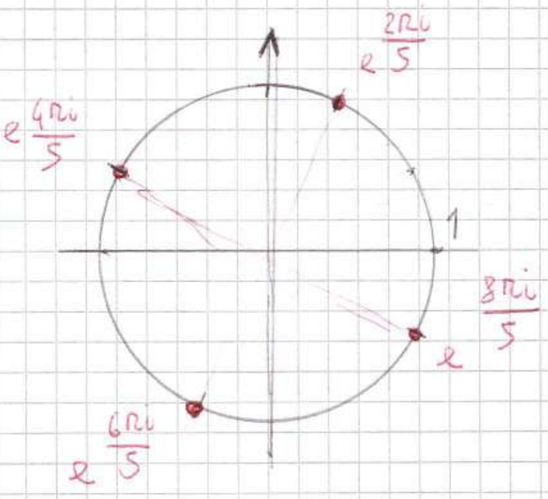
$k=0 \quad \theta=0 \qquad z=1$ da escludere

$k=1 \quad \theta=\frac{2\pi}{5} \qquad \boxed{z_1 = e^{\frac{2\pi i}{5}}}$

$k=2 \quad \theta=\frac{4\pi}{5} \qquad \boxed{z_2 = e^{\frac{4\pi i}{5}}}$

$k=3 \quad \theta=\frac{6\pi}{5} \qquad \boxed{z_3 = e^{\frac{6\pi i}{5}}}$

$k=4 \quad \theta=\frac{8\pi}{5} \qquad \boxed{z_4 = e^{\frac{8\pi i}{5}}}$



Fine.

Test n. ?? – ??

Numeri complessi 4				
Polinomio	z_1	z_2	z_3	z_4
x^2+1	i	$-i$		
x^4-1	1	i	-1	$-i$
x^4+1	$(1+i)/\sqrt{2}$	$(-1+i)/\sqrt{2}$	$(-1-i)/\sqrt{2}$	$(1-i)/\sqrt{2}$
x^4+x^2	0	0	i	$-i$
x^2+2x+1	-1	-1		
x^4-2x^2+1	1	1	-1	-1
x^4+2x^2+1	i	i	$-i$	$-i$
$x^2+4x+13$	$-2+3i$	$-2-3i$		
x^2+ix+6	$2i$	$-3i$		
x^2+x+1	$(-1+i\sqrt{3})/2$	$(-1-i\sqrt{3})/2$		
x^4+x^2+1	$(1+i\sqrt{3})/2$	$(-1+i\sqrt{3})/2$	$(-1-i\sqrt{3})/2$	$(1-i\sqrt{3})/2$
$ix^4+(1+i)x$	0	$(1+i)/\sqrt[3]{2}$	$\sqrt[6]{2}\,e^{11\pi i/12}$	$\sqrt[6]{2}\,e^{19\pi i/12}$
x^4+2x^2+2	$\sqrt[4]{2}\,e^{3\pi i/8}$	$\sqrt[4]{2}\,e^{5\pi i/8}$	$\sqrt[4]{2}\,e^{11\pi i/8}$	$\sqrt[4]{2}\,e^{13\pi i/8}$
x^3-x+2i	i	$-(\sqrt{7}+i)/2$	$(\sqrt{7}-i)/2$	
x^3+x^2+x+1	i	-1	$-i$	
$x^4+x^3+x^2+x+1$	$e^{2\pi i/5}$	$e^{4\pi i/5}$	$e^{6\pi i/5}$	$e^{8\pi i/5}$