Integrali 1

Argomenti: Integrali in una variabile **Difficoltà**: ⋆

Prerequisiti: Primitive elementari

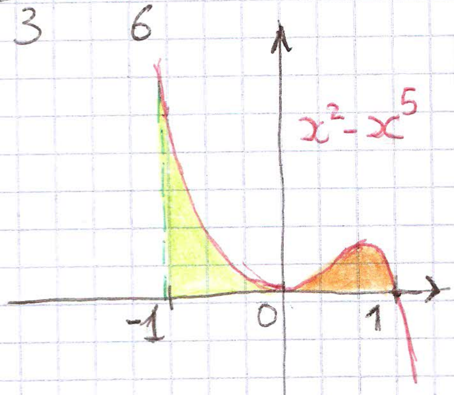
Calcolare l'integrale delle seguenti funzioni sugli insiemi indicati. Indicare "N.P." se l'integranda non è limitata nell'insieme proposto (in questo caso l'esercizio diventa un esercizio sugli integrali impropri) e "N.S." se la richiesta non ha senso.

Funzione	**Ins.**	**Integrale**	**Ins.**	**Integrale**	**Ins.**	**Integrale**
$x^2 - x^5$	$[0,1]$	$1/6$	$[-1,0]$	$1/2$	$[-1,1]$	$2/3$
$\sin x$	$[0,\pi/2]$	1	$[0,\pi]$	2	$[-\pi,\pi]$	0
e^x	$[0,1]$	$e-1$	$[-1,1]$	$e - 1/e$	$[0,\log 7]$	6
e^{-x}	$[0,1]$	$1 - 1/e$	$[-1,1]$	$e - 1/e$	$[0,\log 7]$	$6/7$
2^{-x}	$[0,1]$	$1/\log 4$	$[0,2]$	$3/(4\log 2)$	$[-1,0]$	$1/\log 2$
$\sin(3x)$	$[-\pi,0]$	$-2/3$	$[0,\pi/2]$	$1/3$	$[\pi/3,\pi/2]$	$-1/3$
e^{-6x}	$[0,1]$	$(1-e^{-6})/6$	$[-1,0]$	$(e^6-1)/6$	$[-1,1]$	$(e^6-e^{-6})/6$
$\frac{1}{x^2}$	$[-2,-1]$	$1/2$	$[-1,0]$	N.P. $(+\infty)$	$[1,2]$	$1/2$
$\frac{1}{x}$	$[-1,1]$	NP (Indet.)	$[0,1]$	NP $(+\infty)$	$[-2,-1]$	$-\log 2$
$\frac{1}{1+x^2}$	$[0,1]$	$\pi/4$	$[-1,0]$	$\pi/4$	$[-1,1]$	$\pi/2$
$\sqrt{x}$	$[-1,1]$	N.S	$[0,1]$	$2/3$	$[0,4]$	$16/3$
$\sqrt[3]{x}$	$[-1,1]$	0	$[0,1]$	$3/4$	$[0,8]$	12
$\sqrt{x+3}$	$[0,3]$	$4\sqrt{6} - 2\sqrt{3}$	$[1,6]$	$38/3$	$[6,13]$	$74/3$
$\sqrt{3-x}$	$[0,3]$	$2\sqrt{3}$	$[1,6]$	N.S	$[6,13]$	N.S
$\frac{1}{\sqrt{x}}$	$[-1,1]$	NS	$[0,1]$	NP (2)	$[1,9]$	4
$\frac{1}{\sqrt[4]{x}}$	$[0,1]$	$4/3$	$[1,16]$	$28/3$	$[16,81]$	$76/3$

INTEGRALI 1 Test. 36 Esercizi di A.M. (Parte A)

1) a) $x^2 - x^5$ $[0, 1]$

$$\int_0^1 x^2 - x^5 \, dx = F(x) \text{ t.c } F'(x) = x^2 - x^5 = \frac{x^3}{3} - \frac{x^6}{6}$$



$$\left[\frac{x^3}{3} - \frac{x^6}{6}\right]_0^1 = F(1) - F(0) = \frac{1}{3} - \frac{1}{6} = \boxed{\frac{1}{6}}$$

b) $x^2 - x^5$ $[-1, 0]$

$$\left[\frac{x^3}{3} - \frac{x^6}{6}\right]_{-1}^0 = F(0) - F(-1) = 0 - \left(-\frac{1}{3} - \frac{1}{6}\right) = \boxed{\frac{1}{2}}$$

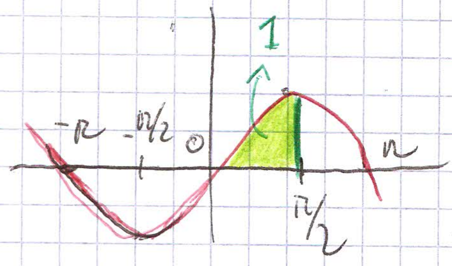
c) $x^2 - x^5$ $[-1, 1]$ l'insieme è l'unione dei primi 2 pertanto l'integrale è $\frac{1}{6} + \frac{1}{2} = \frac{2}{3}$

$$\left[\frac{x^3}{3} - \frac{x^6}{6}\right]_{-1}^1 = F(1) - F(-1) = \frac{1}{6} - \left(-\frac{1}{2}\right) = \frac{2}{3}$$

2) $\sin x \quad [0, \pi/2]$ a

$$\int_0^{\pi/2} \sin x \, dx = F(x) \text{ t.c } F'(x) = \sin x = -\cos x$$

$$\left[-\cos x\right]_0^{\pi/2} = F(\pi/2) - F(0) = 0 - (-1) = \boxed{1}$$



b) $\sin x \quad [0, \pi]$ dal grafico $= 2$

c) $\sin x \quad [-\pi, \pi]$ dal grafico $= 0$

b $\left[-\cos x\right]_0^{\pi} = F(\pi) - F(0) = +1 - (-1) = 2$

c) $\left[-\cos x\right]_{-\pi}^{\pi} = F(\pi) - F(-\pi) = +1 - (+1) = 0$

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3) $e^x \quad [0, 1]$

a $\int_0^1 e^x \, dx = \left[e^x\right]_0^1 = F(1) - F(0) = e - 1$

b $e^x \quad [-1, 1] \quad = \left[e^x\right]_{-1}^{1} = e - e^{-1} = e - \frac{1}{e} = \frac{e^2 - 1}{e}$

c $e^x \quad [0 \quad \log 7] = \left[e^x\right]_0^{\log 7} = e^{\log 7} - 1 = 7 - 1 = 6$

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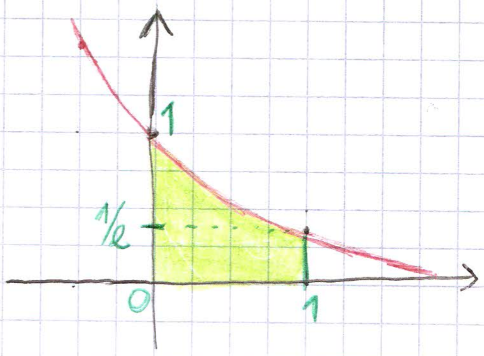
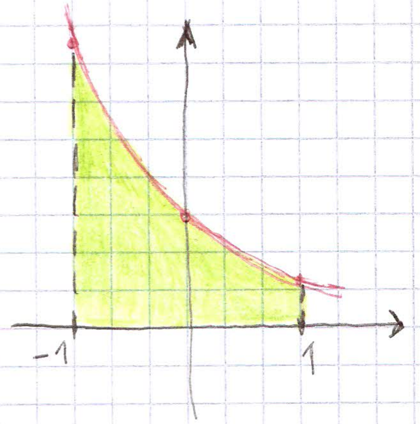
4) a $e^{-x} \quad [0, 1]$

$$\int_0^1 e^{-x} \, dx = F(x) \text{ t.c } F'(x) = e^{-x} = -e^{-x}$$

$$\left[-e^{-x}\right]_0^1 = F(1) - F(0) = -\frac{1}{e} - (-1) = 1 - \frac{1}{e}$$



b) e^{-x} $[-1, 1]$

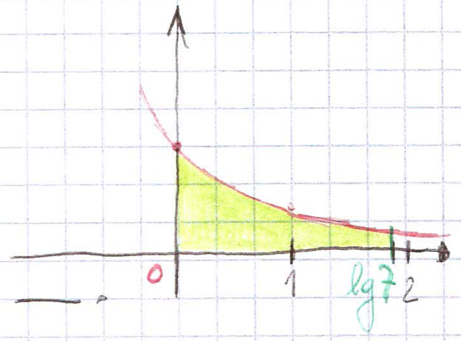


$$\left[-e^{-x}\right]_{-1}^{1} = F(1) - F(-1) = -\frac{1}{e} - (-e) = e - \frac{1}{e}$$

c) e^{-x} $[0 \quad \log 7]$

$$\left[-e^{-x}\right]_{0}^{\log 7} = F(\log 7) - F(0) = -e^{-\log 7} - (-1)$$

$$= -\frac{1}{e^{\log 7}} + 1 = -\frac{1}{7} + 1 = \frac{6}{7}$$



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5) 2^{-x} $[0,1]$

$$\int_0^1 2^{-x}\,dx = F(x) \text{ t.c } F'(x) = 2^{-x} = -\frac{2^{-x}}{\log 2} = F(x)$$

$$\left[F(x)\right]_0^1 = \left[F(x)\right]_{x=a}^{x=b} = F(b) - F(a)$$

$$\left[-\frac{2^{-x}}{\log 2}\right]_0^1 = F(1) - F(0) = -\frac{1}{2\log 2} - \left(-\frac{1}{\log 2}\right) = \frac{1}{2\log 2} = \frac{1}{\log 4}$$

$$\left[-\frac{2^{-x}}{\log 2}\right]_0^2 = F(2) - F(0) = -\frac{1}{4\log 2} - \left(-\frac{1}{\log 2}\right) = \frac{3}{4\log 2}$$

$$\left[-\frac{2^{-x}}{\log 2}\right]_{-1}^0 = F(0) - F(-1) = -\frac{1}{\log 2} - \left(-\frac{2}{\log 2}\right) = \frac{1}{\log 2}$$

6) $\sin 3(x)$ $[-\pi, 0]$

$$\int_{-\pi}^{0} \sin(3x)\,dx = F(x) \quad t.c\ F'(x) = \sin(3x) = -\frac{1}{3}\cos(3x)$$

$$\left[-\frac{\cos(3x)}{3}\right]_{-\pi}^{0} = F(0) - F(-\pi) = -\frac{1}{3} - \left(+\frac{1}{3}\right) = -\frac{2}{3}$$

$$\left[-\frac{\cos(3x)}{3}\right]_{0}^{\pi/2} = F(\pi/2) - F(0) = 0 - \left(-\frac{1}{3}\right) = \frac{1}{3}$$

$$\left[-\frac{\cos 3x}{3}\right]_{\pi/3}^{\pi/2} = F(\pi/2) - F(\pi/3) = 0 - \left(-\frac{(-1)}{3}\right) = -\frac{1}{3}$$

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7) e^{-6x} $[0, 1]$

$$\int_{0}^{1} e^{-6x}\,dx = F(x) \quad t.c\ F'(x) = e^{-6x} \quad F(x) = -\frac{e^{-6x}}{6}$$

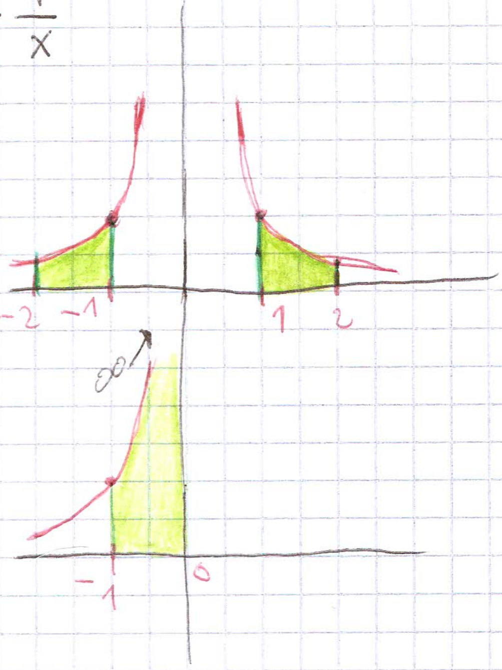
$$\left[-\frac{e^{-6x}}{6}\right]_{0}^{1} = F(1) - F(0) = -\frac{e^{-6}}{6} - \left(-\frac{1}{6}\right) = \frac{1}{6} - \frac{e^{-6}}{6} = \frac{1 - e^{-6}}{6}$$

$$\left[-\frac{e^{-6x}}{6}\right]_{-1}^{0} = F(0) - F(-1) = -\frac{1}{6} - \left(-\frac{e^{6}}{6}\right) = \frac{e^{6}}{6} - \frac{1}{6} = \frac{e^{6} - 1}{6}$$

$$\left[-\frac{e^{-6x}}{6}\right]_{-1}^{1} = F(1) - F(-1) = -\frac{e^{-6}}{6} - \left(-\frac{e^{6}}{6}\right) = \frac{e^{6} - e^{-6}}{6}$$

8) $\frac{1}{x^2}$ $[-2,-1]$, $[-1,0]$ $[1,2]$

$\int_{-1}^{-2} \frac{1}{x^2}\,dx = F(x)$ t.c. $F'(x) = \frac{1}{x^2}$ $\quad F(x) = -\frac{1}{x}$



$\left[-\frac{1}{x}\right]_{-1}^{-2} = F(-1) - F(-2) = 1 - \frac{1}{2} = \frac{1}{2}$

$\left[-\frac{1}{x}\right]_{-1}^{0} = F(0) - F(-1) = N.P.\ \infty$

NON LIMITATA NELL'INSIEME PROPOSTO.

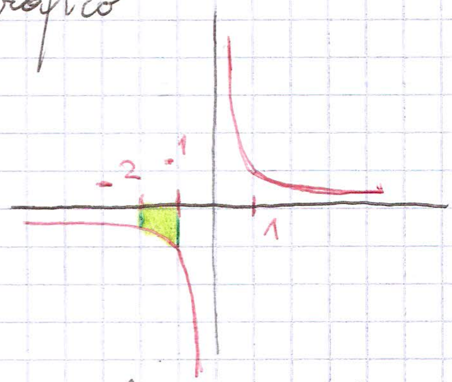
$\left[-\frac{1}{x}\right]_{1}^{2} = F(2) - F(1) = -\frac{1}{2} + 1 = \frac{1}{2}$

9) $\frac{1}{x}$ $[-1,1]$; $[0,1]$; $[-2,-1]$

$\int_{-1}^{1} \frac{1}{x}\,dx = \log|x|$

$\left[\log|x|\right]_{-1}^{1} = F(1) - F(-1) = 0 - 0 \Rightarrow$ NP indeterminata

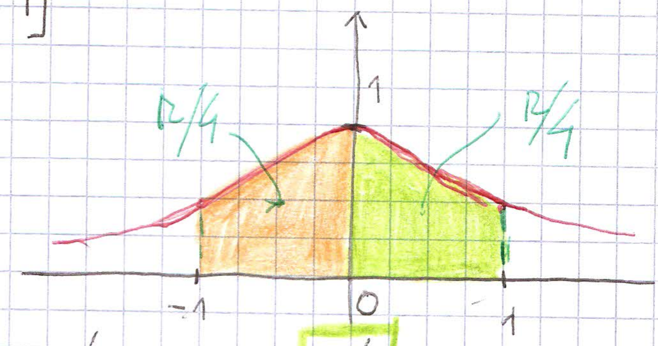
vedere anche il grafico di $\frac{1}{x}$ senza confondersi e "vedere" il grafico di $\log|x|$



$\left[\log|x|\right]_{0}^{1} = N.P\ \infty$

$\left[\log|x|\right]_{-2}^{-1} = F(-1) - F(-2) = \log 1 - \log 2 = 0 - \log 2 = -\log 2$

10) $\frac{1}{1+x^2}$ $[0,1]$; $[-1,0]$ $[-1,1]$



$$\int \frac{1}{1+x^2}\,dx = \arctan x$$

$$\Big[\arctan x\Big]_0^1 = F(1) - F(0) = \arctan 1 - \arctan 0 = \boxed{\pi/4}$$

$$\Big[\arctan x\Big]_{-1}^{0} = F(0) - F(-1) = 0 - (-\pi/4) = \boxed{\pi/4}$$

$$\Big[\arctan x\Big]_{-1}^{1} = F(1) - F(-1) = \frac{\pi}{4} - \left(-\frac{\pi}{4}\right) = \boxed{\pi/2}$$

11) $\sqrt{x}$ $[-1,1]$; $[0,1]$; $[0,4]$

$$\int \sqrt{x}\,dx = F(x) \text{ t.c } F'(x) = \sqrt{x} \Rightarrow F(x) = \frac{2x^{3/2}}{3}$$

$$\left[\frac{2x^{3/2}}{3}\right]_{-1}^{+1} F(1) - F(-1) = \text{N.S per } \sqrt{x}$$

$$\left[\frac{2x^{3/2}}{3}\right]_0^1 = F(1) - F(0) = \frac{2 \cdot 1^{3/2}}{3} = \frac{2}{3}$$

$$\left[\frac{2x^{3/2}}{3}\right]_0^4 = F(4) - F(0) = \frac{2 \cdot \sqrt{4^3}}{3} = \frac{2\sqrt{2^6}}{3} = \frac{2 \cdot 2^3}{3} = \frac{2^4}{3} = \frac{16}{3}$$

12) $\sqrt[3]{x}$ $[-1,1]$; $[0,1]$; $[0,8]$

$$\int \sqrt[3]{x}\,dx = \frac{3x^{4/3}}{4}$$

$$\left[\frac{3x^{4/3}}{4}\right]_{-1}^{1} = 0$$ (pensa al grafico di $\sqrt[3]{x}$) $F(1)-F(-1)=0=\frac{3}{4}-\frac{3}{4}=0$

$$\left[\frac{3x^{4/3}}{4}\right]_{0}^{1} = F(1)-F(0) = \frac{3\cdot 1^{4/3}}{4}-0 = \frac{3}{4}$$

$$\left[\frac{3x^{4/3}}{4}\right]_{0}^{8} = F(8)-F(0) = \frac{3\cdot(2^3)^{4/3}}{2^2} = 12$$

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13) $\sqrt{x+3}$ $[0,3]$; $[1,6]$; $[6,13]$

$$\int \sqrt{x+3}\,dx = \frac{2}{3}(x+3)^{3/2}$$

$$\left[\frac{2}{3}(x+3)^{3/2}\right]_{0}^{3} = F(3)-F(0) = \frac{2}{3}\sqrt{6^3} - \frac{2}{3}\sqrt{3^3}$$

$$= \frac{2}{\cancel{3}}\,\overset{2}{\cancel{6}}\cdot\sqrt{6} - \frac{2\cdot\cancel{3}}{\cancel{3}}\sqrt{3} = 4\sqrt{6}-2\sqrt{3}$$

$$\left[\frac{2}{3}(x+3)^{3/2}\right]_{1}^{6} = F(6)-F(1) = \frac{2}{3}\sqrt{9^3} - \frac{2}{3}\sqrt{4^3} =$$

$$= \frac{2}{3}\cdot 27 - \frac{2}{3}\cdot 8 = 18 - \frac{16}{3} = \frac{38}{3}$$

$$\left[\frac{2}{3}(x+3)^{3/2}\right]_{6}^{13} = \frac{2}{3}\sqrt{16^3} - \frac{2}{3}\sqrt{9^3} = \frac{2}{3}\,64 - \frac{2}{3}\cdot 27 =$$

$$= \frac{128}{3} - 18 = \frac{74}{3}$$

14) $\sqrt{3-x}$ $[0,3]$; $[1,6]$; $[6,13]$

$\sqrt{3-x}$ D $x \leq 3$

$$\int \sqrt{3-x}\, dx = -\frac{2}{3}(3-x)^{3/2}$$

$$\left[-\frac{2}{3}(3-x)^{3/2}\right]_0^3 = F(3) - F(0) = 0 - \left(-\frac{2}{3}\sqrt{3^3}\right) = 2\sqrt{3}$$

$[\,----\,]_1^6$ e $[\,----\,]_6^{13}$ N.S Dominio non fa parte di questi 2 insiemi.

15) $\frac{1}{\sqrt{x}}$ $[-1,1]$; $[0,1]$; $[1,9]$ D = $x > 0$

$$\int \frac{1}{\sqrt{x}}\, dx = 2\sqrt{x}$$

$\left[2\sqrt{x}\right]_{-1}^{-1}$ N.S

$\left[2\sqrt{x}\right]_0^1 = F(1) - F(0) = 2$ N.P x = 0 non è un punto nel dominio della funzione di partenza?

$\left[2\sqrt{x}\right]_1^9 = F(9) - F(1) = 6 - 2 = 4$

16) $\frac{1}{\sqrt[4]{x}}$ $[0,1]$, $[1,16]$; $[16,81]$ D = $x > 0$

$$\int \frac{1}{\sqrt[4]{x}}\, dx = \frac{4}{3}\sqrt[4]{x^3}$$

$$\left[4/3 \cdot \sqrt[4]{x^3}\right]_0^1 = F(1) - F(0) = 4/3$$

$$\left[4/3\sqrt[4]{x^3}\right]_1^{16} = F(16) - F(1) = \frac{4}{3}\sqrt[4]{2^{12}} - \frac{4}{3} = \frac{32}{3} - \frac{4}{3} = \frac{28}{3}$$

$$\left[\frac{4}{3}\sqrt[4]{x^3}\right]_{16}^{81} = F(81) - F(16) = \frac{4}{3}\sqrt[4]{3^{12}} - \frac{4}{3}\sqrt[4]{2^{12}} = 36 - \frac{32}{3} = \frac{76}{3}$$