

$$A = \begin{pmatrix} 3 & 0 & 1 \\ 0 & 3 & 1 \\ 0 & 0 & 3 \end{pmatrix} \quad \lambda_1 = \lambda_2 = \lambda_3 = 3 \quad (A - \lambda I)x = 0 \leadsto \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} x = 0$$

$MA = 3$ $MG = 2$

FORMA \sim JORDAN (2 BLOCCHI)

$$J = \begin{pmatrix} 3 & 1 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{pmatrix} \leadsto \text{BASE JORDANIZZANTE}$$

$$J = M^{-1} A M$$

RICERCA DELLA BASE JORDANIZZANTE

$$M = \begin{pmatrix} | & | & | \\ x_1 & x_2 & x_3 \\ | & | & | \end{pmatrix} \quad M^{-1} A M = J \leadsto A M = M J$$

$$\leadsto A \begin{pmatrix} | & | & | \\ x_1 & x_2 & x_3 \\ | & | & | \end{pmatrix} = \begin{pmatrix} | & | & | \\ x_1 & x_2 & x_3 \\ | & | & | \end{pmatrix} \begin{pmatrix} 3 & 1 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{pmatrix} \leadsto \begin{cases} A x_1 = 3 x_1 \\ A x_2 = x_1 + 3 x_2 \\ A x_3 = 3 x_3 \end{cases}$$

$$A x_1 = 3 x_1 \leadsto (A - 3I) x_1 = 0 \leadsto \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} x_1 = 0 \rightarrow x_1 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$$

$$A x_2 = x_1 + 3 x_2 \leadsto (A - 3I) x_2 = x_1 \leadsto \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} x_2 = x_1 \rightarrow x_2 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$A x_3 = 3 x_3 \leadsto (A - 3I) x_3 = 0 \leadsto \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} x_3 = 0 \quad x_3 = s \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + t \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

$s, t \in \mathbb{R}$

$$\leadsto \begin{pmatrix} | & | & | \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -1 & 0 \end{pmatrix} \begin{pmatrix} | & | & | \\ 3 & 0 & 1 \\ 0 & 3 & 1 \\ 0 & 0 & 3 \end{pmatrix} \begin{pmatrix} | & | & | \\ 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 3 & 1 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{pmatrix} = J$$