

$$f(x, y, z) = xy + z$$

$$x^2 + y^2 + z^2 \leq 1$$

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$$\begin{cases} y = 2\lambda x \\ x = 2\lambda y \\ 1 = 2\lambda z \\ x^2 + y^2 + z^2 = 1 \end{cases} \begin{matrix} \textcircled{1} \\ \textcircled{2} \\ \textcircled{3} \end{matrix} \begin{matrix} \leadsto 2\lambda x^2 = 2\lambda y^2 \xRightarrow{\lambda \neq 0} x = \pm y \quad \lambda = \pm \frac{1}{2} \\ \leadsto z = \pm 1 \\ \leadsto x^2 + y^2 = 0 \Rightarrow x = y = 0 \end{matrix}$$

$$\leadsto \begin{cases} P_1 = (0, 0, 1) \Rightarrow f(P_1) = 1 \quad \text{MAX} \\ P_2 = (0, 0, -1) \Rightarrow f(P_2) = -1 \quad \text{MIN} \end{cases}$$

$$f(x, y, z) = x - y + 3z^2 \quad x^2 + y^2 + z^2 \leq 1$$

$$\begin{cases} 1 = 2\lambda x \\ -1 = 2\lambda y \\ 6z = 2\lambda z \\ x^2 + y^2 + z^2 = 1 \end{cases} \begin{matrix} \textcircled{1} \\ \textcircled{2} \\ \textcircled{3} \end{matrix} \begin{matrix} \leadsto 2\lambda x = -2\lambda y \xRightarrow{\lambda \neq 0} x = -y \\ \leadsto \begin{cases} z = 0 \\ \lambda = 3 \end{cases} \\ \leadsto \begin{cases} z = 0 \Rightarrow x^2 + y^2 = 1 \quad x = \pm \frac{\sqrt{2}}{2} \quad y = \mp \frac{\sqrt{2}}{2} \\ z \neq 0 \Rightarrow x = \frac{1}{6} \quad y = -\frac{1}{6} \quad z = \pm \frac{\sqrt{17}}{3\sqrt{2}} \end{cases} \end{matrix}$$

$$\leadsto \begin{cases} P_1 (\sqrt{2}/2, -\sqrt{2}/2, 0) \Rightarrow f(P_1) = \sqrt{2} \\ P_2 (-\sqrt{2}/2, \sqrt{2}/2, 0) \Rightarrow f(P_2) = -\sqrt{2} \quad \text{MIN} \\ P_3 (1/6, -1/6, \sqrt{17}/3\sqrt{2}) \Rightarrow f(P_3) = \frac{1}{3} + \frac{17}{6} \quad \text{MAX} \\ P_4 (1/6, 1/6, -\sqrt{17}/3\sqrt{2}) \Rightarrow f(P_4) = -\frac{1}{3} + \frac{17}{6} \end{cases}$$

$$f(x, y, z) = y + z - x^2 \quad x^2 + y^2 + z^2 \leq 1$$

$$\begin{cases} -2x = 2\lambda x & \textcircled{2} \rightarrow \begin{cases} x=0 \\ x \neq 0 \Rightarrow \lambda = -1 \end{cases} \\ 1 = 2\lambda y & \textcircled{1} \rightarrow 2\lambda y = 2\lambda z \Rightarrow y = z \\ 1 = 2\lambda z & \lambda \neq 0 \\ x^2 + y^2 + z^2 = 1 & \textcircled{3} \rightarrow \begin{cases} x=0 \Rightarrow y = z = \pm \frac{\sqrt{2}}{2} \\ x \neq 0 \Rightarrow x = \pm \frac{\sqrt{2}}{2} \end{cases} \end{cases}$$

$$\begin{aligned} P_1 &= (0, \sqrt{2}/2, \sqrt{2}/2) \Rightarrow f(P_1) = \sqrt{2} \quad \text{MAX} \\ P_2 &= (0, -\sqrt{2}/2, -\sqrt{2}/2) \Rightarrow f(P_2) = -\sqrt{2} \\ P_3 &= (\sqrt{2}/2, 1/2, 1/2) \Rightarrow f(P_3) = 1/2 \\ P_4 &= (\sqrt{2}/2, -1/2, -1/2) \Rightarrow f(P_4) = -3/2 \quad \text{MIN} \\ P_5 &= (-\sqrt{2}/2, 1/2, 1/2) \Rightarrow f(P_5) = 1/2 \\ P_6 &= (-\sqrt{2}/2, -1/2, -1/2) \Rightarrow f(P_6) = -3/2 \quad \text{MIN} \end{aligned}$$